“Supplementary Material”
Marginal Densities, Factor Graph Duality, and High-Temperature Series Expansions

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1 ADDITIONAL NUMERICAL EXPERIMENTS

We consider a 3-state 2D Potts model with free boundary conditions, with size $N = 6 \times 6$, and in the absence of an external field. The exact marginal densities are computed via the junction tree algorithm implemented in [Mooij, 2010]. In our model all plaquettes (i.e., the cycles of length four) are frustrated. We then focus on the plaquette in the middle of the model.

A plaquette is called frustrated if the product of four coupling parameters along its edges is negative. It is then not possible to satisfy all local constraints at the same time, which leads to difficult energy landscapes [Nishimori, 2001, Weller and Jebara, 2013].

To create frustration, each plaquette has one coupling parameter set to $\beta J_{e_{\mathrm{Antif}}} = -0.25$ (i.e., with antiferromagnetic interaction), and the three remaining couplings set to $\beta J_{e_{\mathrm{Ferr}}}$ according to the value on the x-axis (i.e., with ferromagnetic interaction).

In this example, factors with antiferromagnetic interactions will have negative components in the dual NFG. However, marginal functions can still be estimated via the BP algorithm, c.f. Remark 3 Main Text. (Note that, in principle, BP is applicable to any commutative semiring [Aji and McEliece, 2000].)

Fig. 1 shows the relative error in estimating the marginal density $\pi_{\mathrm{p},e_{\mathrm{Ferr}}}(0)$, (i.e., the edge with ferromagnetic interaction) as a function of $\beta J_{e_{\mathrm{Ferr}}}$. Fig. 1 shows that BP in the dual domain provides the most accurate estimates when $\beta J_{e_{\mathrm{Ferr}}} > 1.90$. For $\beta J_{e_{\mathrm{Ferr}}} < 1.0$, TEP in the primal domain gives the best estimates.

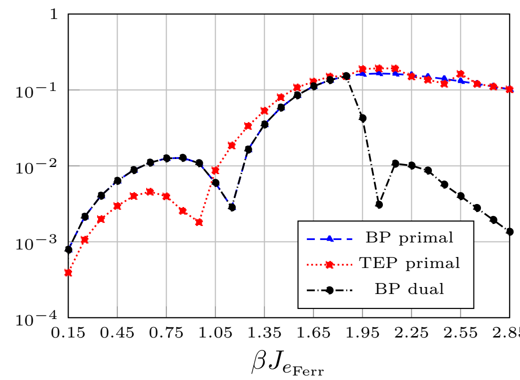

Figure 1: Relative error in estimating the marginal density $\pi_{\mathrm{p},e_{\mathrm{Ferr}}}(0)$ of a frustrated 3-state Potts model with free boundaries and size $N = 6 \times 6$. Here e_{Ferr} has a ferromagnetic interaction $\beta J_{e_{\mathrm{Ferr}}}$.

References

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