Appendix A HMC-CVA Algorithm

We denote the original chain X and its corresponding control variate Y as X^+ and Y^+ for emphasis.

procedure HMC-CVA$(X_0^+, X_0^-, Y_0^+, n, T)$
  for $i = 0$ to $n-1$ **do**
    Sample $p_i^+ \sim \mathcal{N}(0, I_D)$.
    Set $p_i^- = -p_i^+$.
    Set $p_i' = p_i^+$. ▷ Shared p_i
    Set $(q_{i+1}^+, p_{i+1}^+) = \Psi_{H,T}(X_i^+, p_i^+)$.
    Set $(q_{i+1}^-, p_{i+1}^-) = \Psi_{H,T}(X_i^-, p_i^-)$.
    Set $(q_{i+1}', p_{i+1}') = \Psi_{H',T}(Y_i^+, p_i')$.
    Sample $b_i \sim \mathrm{Unif}([0,1])$. ▷ Shared b_i
    Set $X_{i+1}^+ = \mathrm{MHA{\scriptsize DJ}}(X_i^+,\, q_{i+1}^+,\, p_i^+,\, p_{i+1}^+,\, b_i,\, H)$.
    Set $X_{i+1}^- = \mathrm{MHA{\scriptsize DJ}}(X_i^-,\, q_{i+1}^-,\, p_i^-,\, p_{i+1}^-,\, b_i,\, H)$.
    Set $Y_{i+1}^+ = \mathrm{MHA{\scriptsize DJ}}(Y_i^+,\, q_{i+1}',\, p_i',\, p_{i+1}',\, b_i,\, H')$.
    Set $Y_{i+1}^- = 2\mu - Y_{i+1}^+$.
  end for
  Compute unbiased estimate $\hat{\mathbb{E}}_Q[f(Y)]$.
  Estimate optimal β by linear regression.
  for $i = 1$ to n **do**
    Set $Z_{i,j}^+ = f_j(X_i^+) - \beta_j^\top (f(Y_i^+) - \hat{\mathbb{E}}_Q[f(Y)])$.
    Set $Z_{i,j}^- = f_j(X_i^-) - \beta_j^\top (f(Y_i^-) - \hat{\mathbb{E}}_Q[f(Y)])$.
    Set $Z_i = \frac{1}{2}(Z_i^+ + Z_i^-)$.
  end for
end procedure