APPENDIX

A. INTEGRAL IDENTITIES

Let $\Phi_{\mu,\sigma^2}(z)$ be the CDF of a Gaussian distribution with mean μ and variance σ^2. Hence $\Phi_{\mu,\sigma^2}(\log z)$ is the CDF of a log-normal distribution with mean μ and variance σ^2. For some integer K (typically 32 in our experiments), we define I to be the following integral, approximated by the trapezoidal rule:

$$\begin{aligned} I_{\mu,\sigma^2}(y,g) &= \int_0^y \Phi_{\mu,\sigma^2}(\log z)^2 g(z)dz \\ &\approx \sum_{k=0}^{K-1} \frac{1}{2}\left[\Phi_{\mu,\sigma^2}\left(\log z_{k+1}\right)^2 g\left(z_{k+1}\right) + \Phi_{\mu,\sigma^2}\left(\log z_k\right)^2 g\left(z_k\right)\right]\left(z_{k+1}-z_k\right) \end{aligned}$$

where $0 = z_0 < z_1 < ... < z_K = y$ and g is a function. We further define

$$\begin{aligned} I^+_{\mu,\sigma^2}(y) &= I_{\mu,\sigma^2}(y, z \mapsto z), \\ I^-_{\mu,\sigma^2}(y) &= I_{-\mu,\sigma^2}(1/y, z \mapsto 1/z^2). \end{aligned}$$

B. SURVIVAL-CRPS FOR LOG-NORMAL (RIGHT-CENSORED)

For a general continuous prediction distribution F, with actual time to outcome $y \in \mathbb{R}_+$, and censoring indicator c, we generalize the CRPS to the Right Censored Survival CRPS score as:

$$\begin{aligned} \mathcal{S}_{\text{CRPS-RIGHT}}(F,(y,c)) &= \int_{-\infty}^{\infty} (F(z)\mathbb{1}\{z \le \log y \cup c = 0\} - \mathbb{1}\{z \ge \log y \cap c = 0\})^2 dz \\ &= \int_{-\infty}^{\tilde{y}} F(z)^2 dz + (1-c)\int_{\tilde{y}}^{\infty} (F(z)-1)^2 dz. \end{aligned}$$

In the above expression F would generally be in the family of continuous distributions over the entire real line (eg. Gaussian). Alternately, one could also use a family of distributions over the positive reals (e.g log-normal), in which case the Survival CRPS becomes:

$$\begin{aligned} \mathcal{S}_{\text{CRPS-RIGHT}}(F,(y,c)) &= \int_0^{\infty} (F(z)\mathbb{1}\{z \le y \cup c = 0\} - \mathbb{1}\{z \ge y \cap c = 0\})^2 dz \\ &= \int_0^y F(z)^2 dz + (1-c)\int_y^{\infty} (F(z)-1)^2 dz. \end{aligned}$$

For the case of F being log-normal, the expression becomes

$$\begin{aligned} \mathcal{S}_{\text{CRPS-RIGHT}}(F_{\text{LN}(\mu,\sigma^2)},(y,c)) &= \int_0^y \Phi_{\mu,\sigma^2}(\log z)^2 dz + (1-c)\int_y^{\infty} (1-\Phi_{\mu,\sigma^2}(\log z))^2 dz \\ &= \int_0^y \Phi_{\mu,\sigma^2}(\log z)^2 dz + (1-c)\int_y^{\infty} \Phi_{-\mu,\sigma^2}(-\log z)^2 dz \\ &= \int_0^y \Phi_{\mu,\sigma^2}(\log z)^2 dz + (1-c)\int_0^{1/y} \Phi_{-\mu,\sigma^2}(\log z)^2 (1/z)^2 dz \\ &= I^+_{\mu,\sigma^2}(y) + (1-c) I^-_{\mu,\sigma^2}(y). \end{aligned}$$

C. SURVIVAL-CRPS FOR LOG-NORMAL (INTERVAL-CENSORED)

We further extend the Right Censored Survival CRPS to the case of interval censoring. This is particularly useful for all-cause mortality prediction where we assume a particular event must occur by time $\mathcal{T}$. Using the same notations as before, the Interval Censored Survival CRPS is:

$$\mathcal{S}_{\text{CRPS-INTVL}}(F,(y,c,\mathcal{T})) = \int_0^{\infty} (F(z)\mathbb{1}\{\{z \le y \cup c = 0\} \cup z \ge \mathcal{T}\} - \mathbb{1}\{\{z \ge y \cap c = 0\} \cup z \ge \mathcal{T}\})^2 dz$$

$$= \int_0^y F(z)^2 dz + (1-c)\int_y^{\mathcal{T}} (F(z)-1)^2 dz + \int_{\mathcal{T}}^{\infty} (F(z)-1)^2 dz.$$

For the case of F being log-normal, the expression becomes

$$\begin{aligned}
\mathcal{S}_{\text{CRPS-INTVL}}(F_{\text{LN}(\mu,\sigma^2)}, (y,c,\mathcal{T})) &= \int_0^y \Phi_{\mu,\sigma^2}(\log z)^2 dz + (1-c)\int_y^{\mathcal{T}} (1-\Phi_{\mu,\sigma^2}(\log z))^2 dz \\
&\quad + \int_{\mathcal{T}}^{\infty} (1-\Phi_{\mu,\sigma^2}(\log z))^2 dz \\
&= \int_0^y \Phi_{\mu,\sigma^2}(\log z)^2 dz + (1-c)\int_{1/\mathcal{T}}^{1/y} \Phi_{-\mu,\sigma^2}(\log z)^2 (1/z)^2 dz \\
&\quad + \int_0^{1/\mathcal{T}} \Phi_{-\mu,\sigma^2}(\log z)^2 (1/z)^2 dz \\
&= I^+_{\mu,\sigma^2}(y) + I^-_{\mu,\sigma^2}(\mathcal{T}) + (1-c)\Big[I^-_{\mu,\sigma^2}(y) - I^-_{\mu,\sigma^2}(\mathcal{T})\Big].
\end{aligned}$$

D. Survival-AUPRC for log-normal (interval-censored)

We start with the most general case (interval censoring). For a general continuous prediction distribution F with an interval outcome $[L, U]$, we define the Survival-AUPRC as

$$\text{Survival-AUPRC}(F, L, U) = \int_0^1 [F(U/t) - F(Lt)]\, dt.$$

Specifically for the case of log-normal, where ϕ and Φ are PDF and CDF of $\mathcal{N}(0,1)$ respectively, and $\tilde{L} = \log L$ and $\tilde{U} = \log U$:

$$\begin{aligned}
\text{Survival-AUPRC}(F_{\text{LN}(\mu,\sigma^2)}, L, U) &= \int_0^1 \left[F_{\text{LN}(\mu,\sigma^2)}(U/t) - F_{\text{LN}(\mu,\sigma^2)}(Lt)\right] dt \\
&= \int_0^1 \Big[F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U} - \log t) - F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L} + \log t)\Big] dt \\
(\text{substituting } s = \log t) &= \int_{-\infty}^0 \Big[F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U} - s) - F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L} + s)\Big] e^s ds \\
&= \Big[F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U} - s) - F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L} + s)\Big] e^s \Big|_{s=-\infty}^{s=0} \\
&\quad - \int_{-\infty}^0 \Big[-f_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U} - s) - f_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L} + s)\Big] e^s ds \\
&= \Big(F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U}) - F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L})\Big) \\
&\quad + \int_{-\infty}^0 \Big[f_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U} - s) + f_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L} + s)\Big] e^s ds \\
&= \Big(F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U}) - F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L})\Big) \\
&\quad + \int_{-\infty}^0 f_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U} - s) e^s ds + \int_{-\infty}^0 f_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L} + s) e^s ds \\
&= \Big(F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U}) - F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L})\Big) \\
&\quad + \int_{-\infty}^0 \frac{1}{\sigma}\phi\left(\frac{\tilde{U} - s - \mu}{\sigma}\right) e^s ds + \int_{-\infty}^0 \frac{1}{\sigma}\phi\left(\frac{\tilde{L} + s - \mu}{\sigma}\right) e^s ds \\
\left(\text{substituting } u = \frac{\tilde{U} - s - \mu}{\sigma}\right) &= \Big(F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U}) - F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L})\Big)
\end{aligned}$$

$$
\begin{aligned}
&+ \int_{\infty}^{\frac{\tilde{U}-\mu}{\sigma}} \frac{1}{\sigma}\phi\left(u\right) e^{\tilde{U}-\sigma u-\mu}(-\sigma)du + \int_{-\infty}^{0} \frac{1}{\sigma}\phi\left(\frac{\tilde{L}+s-\mu}{\sigma}\right) e^{s}ds \\
\left(\text{substituting } v = \frac{\tilde{L}+s-\mu}{\sigma}\right) &= \left(F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U}) - F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L})\right) \\
&+ \int_{\infty}^{\frac{\tilde{U}-\mu}{\sigma}} \frac{1}{\sigma}\phi\left(u\right) e^{\tilde{U}-\sigma u-\mu}(-\sigma)du + \int_{-\infty}^{\frac{\tilde{L}-\mu}{\sigma}} \frac{1}{\sigma}\phi\left(v\right) e^{v\sigma-\tilde{L}+\mu}\sigma dv \\
&= \left(F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U}) - F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L})\right) \\
&- e^{\tilde{U}-\mu}\int_{\infty}^{\frac{\tilde{U}-\mu}{\sigma}} \phi\left(u\right) e^{-\sigma u}du + e^{-\tilde{L}+\mu}\int_{-\infty}^{\frac{\tilde{L}-\mu}{\sigma}} \phi\left(v\right) e^{v\sigma}dv \\
\left(\text{using } \int e^{cx}\phi(x)dx = e^{\frac{c^2}{2}}\Phi(x-c)\right) &= \left(F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U}) - F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L})\right) \\
&+ \frac{U}{e^{\mu}}\left[e^{\frac{\sigma^2}{2}}\Phi(u+\sigma)\right]_{u=\infty}^{u=\frac{\tilde{U}-\mu}{\sigma}} + \frac{e^{\mu}}{L}\left[e^{\frac{\sigma^2}{2}}\Phi(v-\sigma)\right]_{v=-\infty}^{v=\frac{\tilde{L}-\mu}{\sigma}} \\
&= \left(F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U}) - F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L})\right) \\
&+ \frac{U}{e^{\mu}}\left[e^{\frac{\sigma^2}{2}}\Phi\left(\frac{\tilde{U}-\mu}{\sigma}+\sigma\right) - e^{\frac{\sigma^2}{2}}\right] + \frac{e^{\mu}}{L}\left[e^{\frac{\sigma^2}{2}}\Phi\left(\frac{\tilde{L}-\mu}{\sigma}-\sigma\right)\right] \\
&= \left(F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{U}) - F_{\mathcal{N}(\mu,\sigma^2)}(\tilde{L})\right) \\
&+ e^{\frac{\sigma^2}{2}}\left[\frac{e^{\mu}}{L}\Phi\left(\frac{\tilde{L}-\mu}{\sigma}-\sigma\right) + \frac{U}{e^{\mu}}\left(1-\Phi\left(\frac{\tilde{U}-\mu}{\sigma}+\sigma\right)\right)\right] \\
&= \Phi_{\mu,\sigma^2}(\log U) - \Phi_{\mu,\sigma^2}(\log L) \\
&+ e^{\frac{\sigma^2}{2}}\left[\frac{e^{\mu}}{L}\Phi\left(\frac{\log L-\mu}{\sigma}-\sigma\right) + \frac{U}{e^{\mu}}\Phi\left(-\frac{\log U-\mu}{\sigma}-\sigma\right)\right].
\end{aligned}
$$

E. Survival-AUPRC for log-normal (right-censored)

For a general continuous prediction distribution F with an interval outcome $[L, \infty)$, we define Survival-AUPRC as

$$
\text{Survival-AUPRC}(F, L) = \int_0^1 \left[1 - F(Lt)\right] dt.
$$

Specifically for the case of log-normal, where Φ is the CDF of $\mathcal{N}(0,1)$, and $\tilde{L} = \log L$ (following Appendix-D),

$$
\text{Survival-AUPRC}(F_{\text{LN}(\mu,\sigma^2)}, L) = \int_0^1 \left[1 - F_{\text{LN}(\mu,\sigma^2)}(Lt)\right] dt = 1 - \Phi_{\mu,\sigma^2}(\tilde{L}) + \frac{e^{\mu+\frac{\sigma^2}{2}}}{L}\Phi\left(\frac{\tilde{L}-\mu}{\sigma}-\sigma\right).
$$

F. Survival-AUPRC for log-normal (uncensored)

For a general continuous prediction distribution F with a point outcome y, we define Survival-AUPRC

$$
\text{Survival-AUPRC}(F, y) = \int_0^1 \left[F(y/t) - F(yt)\right] dt.
$$

Specifically for the case of log-normal, where Φ is the CDF of $\mathcal{N}(0,1)$, and $\tilde{y} = \log y$ (following Appendix-D),

$$
\begin{aligned}
\text{Survival-AUPRC}(F_{\text{LN}(\mu,\sigma^2)}, y) &= \int_0^1 \left[F_{\text{LN}(\mu,\sigma^2)}(y/t) - F_{\text{LN}(\mu,\sigma^2)}(yt)\right] dt \\
&= e^{\frac{\sigma^2}{2}}\left[\frac{e^{\mu}}{y}\Phi\left(\frac{\tilde{y}-\mu}{\sigma}-\sigma\right) + \frac{y}{e^{\mu}}\Phi\left(-\frac{\tilde{y}-\mu}{\sigma}-\sigma\right)\right].
\end{aligned}
$$

G. Evaluation as binary outcome

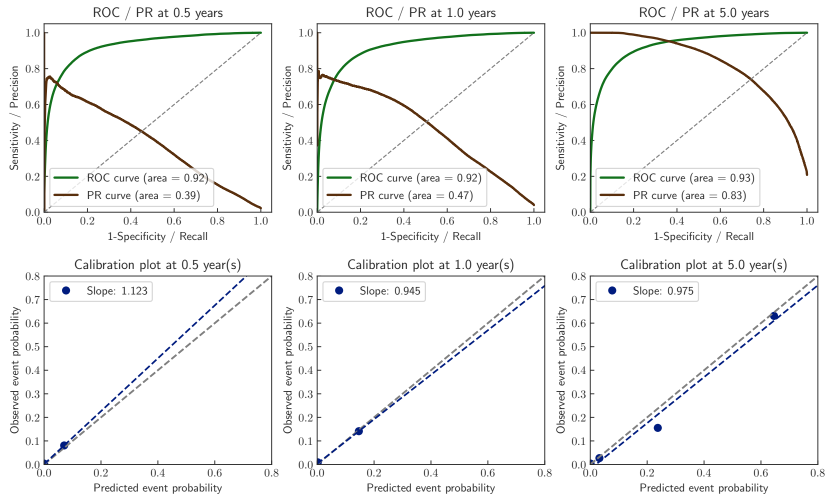

Figure 6: Discrimination and calibration of predictions from the interval-censored Survival-CRPS model, evaluated as predictions for a dichotomous outcome at 6 months, 1 year, and 5 years.

H. Individual Patients in Interval-Censored Survival-CRPS Model

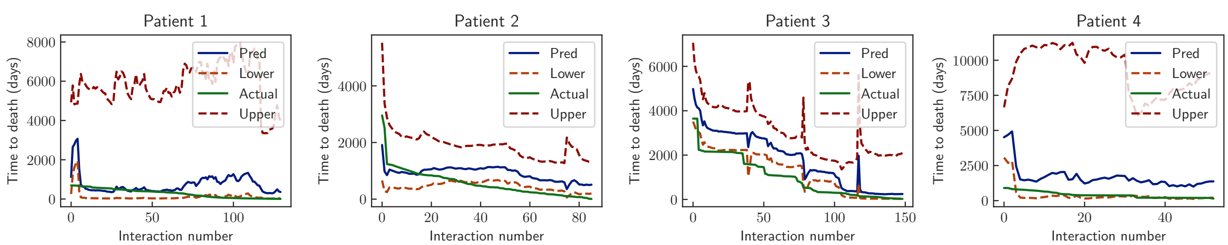

Figure 7: Median predicted time to death (with 95% intervals) for individual patients from the interval-censored Survival-CRPS model. Our model gives more confident predictions upon repeated interactions between patients and the EHR. True times to death generally lie within predicted intervals.