A. Appendices

A.1. Proof of Proposition 1

Proof. It could be shown that a DAG D is a member of MEC corresponding to G if and only if it has no v-structures (He et al., 2015). Let $v \in V$ be an arbitrary node. For every DAG in the MEC, the parent set of node v is definitely a clique, because a v-structure is formed otherwise. If D_1 and D_2 be two members of MEC such that $pa_{D_1}(v) = pa_{D_2}(v)$ then $D_1 \sim_{\{\{v\}\}} D_2$, and therefore D_1 and D_2 are indistinguishable under the single-node intervention target $\{v\}$ (Hauser & Bühlmann, 2014). So every $\mathcal{E}_{\{\{v\}\}}(D)$ is determined uniquely with $pa_D(v)$. On the other hand, Every LexBFS-ordering σ on G, is also a perfect elimination ordering and if we orient edges of G according to σ, we get a DAG without v-structures (Hauser & Bühlmann, 2014). For an arbitrary clique $P \subseteq ne_G(v)$ in neighbors of v, if we orient edge set E according to $LexBFS((P, v, ...), E)$, the resulting DAG D is a member of MEC and $pa_D(v) = P$. This shows that there is a one-to-one correspondence between $\mathcal{E}_{\{\{v\}\}}(D)$s and cliques $P \subseteq ne_G(v)$. □

A.2. Proof of Theorem 1

Proof. The proofs of four statements is respectively as follows:

- Every node which is separated from v by P is inside A_R, so for every $d \in D_R$ there is a path from d to v in $G[V \backslash A_R]$. Now assume that there is an edge between two arbitrary nodes $a \in A_R$ and $d \in D_R$. As there is a path from v to d in $G[V \backslash A_R]$, and edge $a - d$ is also present in $G[V \backslash A_R]$, there is a path from v to a in $G[V \backslash A_R]$ and therefore P is not an (a, v)-separator in G, which could not be true.

- The cycle $a \to v \to b \to a$ is formed otherwise.

- If $a \in C_R$ and the edge be directed as $b \to a$, the v-structure $v \to a \leftarrow b$ will be formed. If $a \in P$, let $v, x_1, x_2, ..., x_k, b$ be the shortest path between v and b in $G[\{v\} \cup C_R \cup D_R]$. No two non-consecutive nodes of this path are connected to each other, because we will find a shorter path otherwise. It is also obvious that $x_1 \in C_R$ and therefore $v \to x_1 \in R$. If $x_1 - x_2$ be directed as $x_1 \leftarrow x_2$ in R, the v-structure $v \to x_1 \leftarrow x_2$ will be formed, so $x_1 \to x_2 \in R$. With a similar arguement, we can say $x_i \to x_{i+1} \in R$, for $1 \leq i \leq k$, where $x_{k+1} = b$. Therefore $v \to x_1 \to x_2 \to ... \to x_k \to b$ is a directed path in R. If $b \to a \in R$, we will have a cycle in R which is impossible, and therefore $a \to b \in R$.

- None of the edges inside $R[A_R \cup P]$ are oriented as a direct result of intervention, so every edge in this subgraph should be oriented using Meek rules. Let $a \to b$ be the first edge oriented inside $R[A_R \cup P]$, so we have $a, b \in A_R \cup P$. In all of the four Meek rules, there is at least one already oriented edge directed towards one of the two endpoints of the edge which is being oriented. This means that there should exist either an edge $x \to a \in R$ or and edge $x \to b \in R$. But this is impossible, because we know that there are no edges directed towards any of the nodes in $A_R \cup P$ in the graph we get after intervention. This means that no Meek rules are applicable for orienting edges in $R[A_R \cup P]$, and this subgraph is undirected.

□

A.3. Proof of Lemma 1

We break the lemma into two smaller lemmas and prove them separately:

Lemma 3. *Let $G(V, E)$ be a UCCG and $\mathcal{I}$ be an arbitrary intervention family. Then we have:*

$$|MEC(G)| = \sum_{R \in \mathcal{IR}_{\mathcal{I}}(G)} |MEC(R)|.$$

Proof. Every DAG D in MEC corresponding to G is exactly in one of the $\mathcal{I}$-essential graphs in $\mathcal{IR}(G)$, based on direction of the edges connected to intervention targets inside that DAG. Therefore, each DAG is exactly counted once in the summation. □

Lemma 4. *Consider $\mathcal{I}$-essential graph $\mathcal{E}_{\mathcal{I}}(D)$ of a DAG D and intervention target $\mathcal{I}$. Let $\mathcal{C}(\mathcal{E}_{\mathcal{I}}(D))$ be the set of all chain components of $\mathcal{E}_{\mathcal{I}}(D)$. Then we have:*

$$|MEC(\mathcal{E}_{\mathcal{I}}(D))| = \prod_{C \in \mathcal{C}(\mathcal{E}_{\mathcal{I}}(D))} |MEC(C)|.$$

Proof. (Hauser & Bühlmann, 2014) showed that the direction of edges inside each chain component of an $\mathcal{I}$-essential graph is unrelated to the direction of edges in other components. Therefore edges inside each chain component could be oriented independently, and number of valid orientations of edges in $\mathcal{E}_{\mathcal{I}}(D)$ (orientations without v-structures) is equal to multiplication of number of valid orientations in each chain component. He et al. (2015) proved a similar lemma for observational cases. □

Lemma 3 shows that we can calculate the size of MEC represented by G via calculating sizes of $\mathcal{I}$-MECs represented by members of $\mathcal{IR}_{\{\{v\}\}}(G)$. For counting number of DAGs in each of these $\mathcal{I}$-MECs, we use Lemma 4, and therefore the equation in Lemma 1 holds.

A.4. Proof of Lemma 2

We need these two lemmas for the proof:

Lemma 5. *(Ghassami et al., 2018) For any DAG $D(V, E)$ and sets $I_1, I_2 \subseteq V$, we have:*

$$Dir(\mathcal{E}_{\{I_1 \cup I_2\}}(D)) = Dir(\mathcal{E}_{\{I_1\}}(D)) \cup Dir(\mathcal{E}_{\{I_2\}}(D)).$$

Lemma 6. *(Hauser & Bühlmann, 2014) Consider an $\mathcal{I}$-essential graph of some DAG D, and let $C \in \mathcal{C}(\mathcal{E}_{\mathcal{I}}(D))$ be one of its chain components. Let $I \subseteq V, I \notin \mathcal{I}$ be another intervention target. Then we have:*

$$\mathcal{E}_{\mathcal{I}\cup\{I\}}(D)[C] = \mathcal{E}_{\{\emptyset, I\cap V'\}}(D[C])$$

Now we prove Lemma 2.

Proof. Using Lemma 6 we can say:

$$\mathcal{E}_{\{\{v_1\},\{v_2\},\ldots,\{v_k\}\}}(D)[C] =$$

$$\mathcal{E}_{\{\{v_1\}\}\cup\{\{v_2\},\ldots,\{v_k\}\}}(D)[C] =$$

$$\mathcal{E}_{\{\emptyset,\{\{v_2\},\ldots,\{v_k\}\}\cap V'\}}(D[C]) =$$

$$\mathcal{E}_{\{\{\{v_2\},\ldots,\{v_k\}\}\cap V'\}}(D[C]) =$$

$$\mathcal{E}_{\{\{v_1^C\},\ldots,\{v_m^C\}\}}(D[C])$$

Where the equality between third and fourth lines comes from the fact that we already know the observational essential graph of the chain component, as we are given the UCCG. Using Lemma 5, we have:

$$Dir(\mathcal{E}_{\{\{v_1\},\{v_2\},\ldots,\{v_k\}\}}(D))$$

$$= Dir(\mathcal{E}_{\{\{v_1\}\}}(D)) \cup Dir(\mathcal{E}_{\{\{v_2\},\{v_3\},\ldots,\{v_k\}\}}(D))$$

But as we mentioned earlier, direction of edges inside one chain component gives us no information about direction of edges in other chain components.

We can say:

$$\begin{aligned} & Dir(R_1) \cup Dir(\mathcal{E}_{\{\{v_2\},\{v_3\},\ldots,\{v_k\}\}}(D)) \\ &= Dir(R_1) \bigcup_{C\in\mathcal{C}(R_1)} Dir(\mathcal{E}_{\{\{v_2\},\{v_3\},\ldots,\{v_k\}\}}(D)[C]) \\ &= Dir(R_1) \bigcup_{C\in\mathcal{C}(R_1)} Dir(\mathcal{E}_{\{\{v_1^C\},\{v_2^C\},\ldots,\{v_{m_C}^i\}\}}(D)[C]). \end{aligned}$$

□

A.5. Proof of Proposition 2

Proof. We know that every valid orientation of all undirected edges in all of the chain components gives us a DAG in the $\mathcal{I}$-MEC $\mathcal{E}_{\{\{v_1\}\}}(D)$. Moreover we know that the minimum value of $|Dir(\mathcal{E}_{\{\{v_1^C\},\{v_2^C\},\ldots,\{v_{m_C}^C\}\}}(D)[C])|$ is $DP[C][\{v_1^C, v_2^C, \ldots, v_{m_C}^C\}] = DP[C][T \cap C]$. As chain components have distinct edges sets, we have:

$$\Big| \bigcup_{C\in\mathcal{C}(\mathcal{E}_{\{\{v_1\}\}}(D))} Dir(\mathcal{E}_{\{\{v_1^C\},\{v_2^C\},\ldots,\{v_{m_C}^C\}\}}(D[C])) \Big| = \sum_{C\in\mathcal{C}(\mathcal{E}_{\{\{v_1\}\}}(D))} \Big| Dir(\mathcal{E}_{\{\{v_1^C\},\{v_2^C\},\ldots,\{v_{m_C}^C\}\}}(D[C])) \Big|.$$

Lemma 2 implies that for counting number of directed edges in each $\mathcal{I}$-essential graph, we could consider each component independently and therefore the minimum number of directed edges for each chain component can be found via DP values. We can iterate over all possible $\mathcal{E}_{\{\{v_1\}\}}(D)$s and use DP values to find the minimum number of directed edges for each case. This means $DP[V][T]$ could be calculated by the recursive formula (4). □