Appendix A. Derivation

We leverage the negative logarithmic probability to measure the differences between the original \( x^{(2)} \) and the reconstructed one. Let \( R(x^{(2)}) \) denote the event that after passing the loop \( S_2 \rightarrow S_k \rightarrow S_1 \rightarrow S_2 \), \( x^{(2)} \) is reconstructed to \( x^{(2)} \). We have that

\[
\ln \Pr(R(x^{(2)})) = \sum_{x^{(k)} \in S_k} \sum_{x^{(1)} \in S_1} \ln \Pr(x^{(2)}, x^{(1)}, x^{(k)} | \quad \text{starting from } x^{(2)}, \text{applied by } \theta_{2k}, \theta_{k1}, \theta_{12} \text{ sequentially})
\]

\[
= \sum_{x^{(k)} \in S_k} \sum_{x^{(1)} \in S_1} \ln \left( \Pr(x^{(1)}, x^{(k)} | x^{(2)}; \theta_{2k}, \theta_{k1}) \cdot \Pr(x^{(2)} | x^{(1)}; \theta_{12}) \right)
\]

\[
\geq \sum_{x^{(k)} \in S_k} \sum_{x^{(1)} \in S_1} \Pr(x^{(1)}, x^{(k)} | x^{(2)}; \theta_{2k}, \theta_{k1}) \cdot \ln \Pr(x^{(2)} | x^{(1)}; \theta_{12})
\]

\[
= \sum_{x^{(k)} \in S_k} \sum_{x^{(1)} \in S_1} \Pr(x^{(k)} | x^{(2)}; \theta_{2k}, \theta_{k1}) \Pr(x^{(1)} | x^{(k)}; \theta_{2k}, \theta_{k1}) \cdot \ln \Pr(x^{(2)} | x^{(1)}; \theta_{12})
\]

\[
= \mathbb{E}_{x^{(k)} \sim \Pr(\cdot | x^{(2)}; \theta_{2k})} \mathbb{E}_{x^{(1)} \sim \Pr(\cdot | x^{(k)}; \theta_{k1})} \ln \Pr(x^{(2)} | x^{(1)}; \theta_{12}).
\]

In Eqn. (1), the first \( \Pr \) represents the jointly probability that \( x^{(2)} \) can be translated into \( x^{(k)} \) with \( \theta_{2k} \), and the the obtained \( x^{(k)} \) can be translated into \( x^{(1)} \) with \( \theta_{k1} \); the second \( \Pr \) represents the probability that given \( x^{(1)} \), it can be translated back to \( x^{(2)} \) with \( \theta_{12} \).