A Omitted proofs

A.1 Omitted proof of Lemma 4 (mixing coefficient)

We finish the proof of lemma 4 by proving $\beta_{S_X}(2i\eta) = \beta_{L_{\tilde{X}}}(2i\eta)$:

$$\begin{aligned}
\beta_{S_X}(2i\eta) = \frac{1}{2}\int \Big| & \pi(\boldsymbol{x}_0, \boldsymbol{x}'_0)\pi(\boldsymbol{x}_{2i\eta}, \boldsymbol{x}'_{2i\eta}) \\
& - \pi(\boldsymbol{x}_0, \boldsymbol{x}'_0) p(\boldsymbol{x}_{2i\eta}, \boldsymbol{x}'_{2i\eta} | \boldsymbol{x}_0, \boldsymbol{x}'_0)\Big| \\
= & \frac{1}{2}\int \pi(\boldsymbol{x}_0, \boldsymbol{x}'_0) \cdot \big|\pi(\boldsymbol{x}_{2i\eta}, \boldsymbol{x}'_{2i\eta}) - p(\boldsymbol{x}_{2i\eta}, \boldsymbol{x}'_{2i\eta} | \boldsymbol{x}_0, \boldsymbol{x}'_0)\big| \\
= & \frac{1}{2}\int \left(\frac{1}{2}\pi(\boldsymbol{x}_0)\left(p(\boldsymbol{x}'_0|\boldsymbol{x}_0) + \pi(\boldsymbol{x}'_0)\right)\right) \\
& \cdot \left(\frac{1}{2}\big|\pi(\boldsymbol{x}_{2i\eta}) - p(\boldsymbol{x}_{2i\eta}|\boldsymbol{x}_0)\big|\left(p(\boldsymbol{x}'_{2i\eta}|\boldsymbol{x}_{2i\eta}) + \pi(\boldsymbol{x}'_{2i\eta})\right)\right) \\
= & \frac{1}{8}\int_{\boldsymbol{x}_0, \boldsymbol{x}_{2i\eta}} \pi(\boldsymbol{x}_0)\big|\pi(\boldsymbol{x}_{2i\eta}) - p(\boldsymbol{x}_{2i\eta}|\boldsymbol{x}_0)\big| \\
& \cdot \int_{\boldsymbol{x}'_0} \left(p(\boldsymbol{x}'_0|\boldsymbol{x}_0) + \pi(\boldsymbol{x}'_0)\right) \cdot \int_{\boldsymbol{x}'_{2i\eta}} \left(p(\boldsymbol{x}'_{2i\eta}|\boldsymbol{x}_{2i\eta}) + \pi(\boldsymbol{x}'_{2i\eta})\right) \\
= & \frac{1}{2}\int_{\boldsymbol{x}_0, \boldsymbol{x}_{2i\eta}} \pi(\boldsymbol{x}_0)\big|\pi(\boldsymbol{x}_{2i\eta}) - p(\boldsymbol{x}_{2i\eta}|\boldsymbol{x}_0)\big| \\
= & \frac{1}{2}\int_{\tilde{\boldsymbol{x}}_0, \tilde{\boldsymbol{x}}_{2i\eta}} \pi(\tilde{\boldsymbol{x}}_0)\big|\pi(\tilde{\boldsymbol{x}}_{2i\eta}) - p(\tilde{\boldsymbol{x}}_{2i\eta}|\boldsymbol{x}_0)\big| = \beta_{L_{\tilde{X}}}(2i\eta).
\end{aligned} \tag{32}$$

A.2 Omitted calculations for Theorem 1 (sample complexity)

We now provide the calculation details for Theorem 1.

By lemma 2, and recall the empirical Rademacher complexity is $\mathfrak{R}_\mu = O\left(K_\eta(\mathcal{H})\sqrt{\log\mu/\mu}\right)$, we need to choose T, μ such that

$$C\sqrt{\frac{1}{\mu}}\left(K_\eta(\mathcal{H})\sqrt{\log\mu} + \sqrt{-\log\left(\delta - \Delta_{appr}\right)}\right) \le \Delta_{gen} \tag{33}$$

where $\Delta_{appr}^\mu := O\left(\frac{B}{\sqrt{T}}\mu^{\frac{3}{2}}\right)$ by lemma 3.

We would like to control $\Delta_{appr}^\mu = O(\delta)$. Substituting in the choice of $T = \Omega\left(\frac{B^2 K_\eta(\mathcal{H})^3}{\delta^2\Delta_{gen}^3}\left(\log\frac{1}{\delta}\right)^{\frac{3}{2}}\right)$, we have

$$\Delta_{appr}^\mu = O\left(\frac{B\delta\Delta_{gen}^{3/2}}{BK_\eta(\mathcal{H})^{3/2}}\left(\log\frac{1}{\delta}\right)^{-3/2}\cdot\mu^{3/2}\right) = O(\delta) \tag{34}$$

which is satisfied by setting $\mu = \Theta\left(\frac{K_\eta(\mathcal{H})\sqrt{\log(1/(\delta-\Delta_{appr}))}}{\Delta_{gen}}\right)$.

A.3 Omitted calculations of Theorem 3 (guarantee on p^η)

Recall that c_*, C_* and $\hat{c}, \hat{C}$ are the constants in lemma 6 for p_*^η and p^η respectively. Denote $c := \max\{c_*, \hat{c}\}$, $C_u = \max\{C_*, \hat{C}\}$, $C_l = \min\{C_*, \hat{C}\}$.

We now show the omitted calculations for equation 31 in the proof of Theorem 3.

$$
\begin{aligned}
&\mathbb{E}_{\boldsymbol{x},\boldsymbol{x}'}\frac{\left(\max\{p^{\eta},p^{\eta}_{*}\}+q\right)^{4}}{q^{2}}\\
\leq&\mathbb{E}_{\boldsymbol{x},\boldsymbol{x}'}Z_{\sigma}^{2}\exp\left(\frac{1}{\sigma^{2}}\|\boldsymbol{x}-\boldsymbol{x}'\|^{2}\right)\cdot\left(\frac{c}{\eta^{d/2}}\exp(C_{u}\eta\|\boldsymbol{x}\|^{2})+\frac{1}{Z_{\sigma}}\right)^{4}\exp\left(-\frac{4}{C_{l}\eta}\|\boldsymbol{x}-\boldsymbol{x}'\|^{2}\right)\\
\leq&\mathbb{E}_{\boldsymbol{x}}Z_{\sigma}^{2}\left(\frac{c}{\eta^{d/2}}\exp(C_{u}\eta\|\boldsymbol{x}\|^{2})+\frac{1}{Z_{\sigma}}\right)^{4}\mathbb{E}_{\boldsymbol{x}'}\exp\left(-\frac{2}{C_{l}\eta}\|\boldsymbol{x}-\boldsymbol{x}'\|^{2}\right)\\
\leq&\mathbb{E}_{\boldsymbol{x}}\frac{cZ_{\sigma}^{2}}{\eta^{d/2}}\left(\frac{c}{\eta^{d/2}}\exp(C_{1}\eta\|\boldsymbol{x}\|^{2})+(\pi C_{l}\eta)^{-\frac{d}{2}}\right)^{4}\exp\left(C_{u}\eta\|\boldsymbol{x}\|^{2}\right)\int_{\boldsymbol{x}'}\exp\left(-\frac{3}{C_{l}\eta}\|\boldsymbol{x}-\boldsymbol{x}'\|^{2}\right)\\
\leq&\frac{cZ_{\sigma}^{2}}{\eta^{5d/2}}\left(\frac{2\pi C_{l}\eta}{3}\right)^{\frac{d}{2}}\mathbb{E}_{\boldsymbol{x}}\left(c\exp(C_{u}\eta\|\boldsymbol{x}\|^{2})+(\pi C_{l})^{-\frac{d}{2}}\right)^{4}\exp\left(C_{u}\eta\|\boldsymbol{x}\|^{2}\right)\\
\leq&c\left(\frac{2\pi^{3}C_{2}^{3}}{\eta^{2}}\right)^{d/2}\exp(-f(\boldsymbol{x}_{*}))\int_{\boldsymbol{x}}\left(c\exp(C_{u}\eta\|\boldsymbol{x}\|^{2})+(\pi C_{l})^{-\frac{d}{2}}\right)^{4}\exp\left(-\left(\frac{\rho}{2}-C_{u}\eta\right)\|\boldsymbol{x}\|^{2}\right)\\
\leq&16c\pi(\boldsymbol{x}_{*})\left(\frac{2\pi^{3}C_{l}^{3}}{\eta^{2}}\right)^{d/2}\left[c^{4}\left(\frac{\rho}{2}-5C_{1}\eta\right)^{-\frac{d}{2}}+(\pi C_{l})^{-2d}\left(\frac{\rho}{2}-C_{u}\eta\right)^{-\frac{d}{2}}\right]\\
\leq&O\left(\pi(\boldsymbol{x}_{*})\left(\frac{1}{\rho\eta^{2}}\right)^{d/2}\right).
\end{aligned}
\tag{35}
$$

where C_1, C_2 are constants introduced to simplify the notations.