A Proof of Theorem 4.1

Let a stationary point $\pi \in \mathbb{R}_+^n$ of the Augmented Lagrangian (4) be such that:

$$\frac{\partial L_\rho(\pi, W^k, y^k)}{\partial \pi_i} = 0 \quad \forall i \in [n]$$

\[ \text{(20a)} \]

$$\iff \frac{\partial L(D|\pi)}{\partial \pi_i} + y^k_i + \rho \frac{\partial D_\rho(\pi||\bar{\pi})}{\partial \pi_i} = 0, \quad \forall i \in [n]$$

\[ \text{(20b)} \]

Let $\sigma_i(\pi) = \rho \frac{\partial D_\rho(\pi||\bar{\pi})}{\partial \pi_i} + y^k_i$, for all $i \in [n]$. Then, Eq. (20a) is equivalent to:

$$\frac{\partial L(D|\pi)}{\partial \pi_i} + \sigma_i(\pi) = 0 \quad \forall i \in [n].$$

\[ \text{(21)} \]

Partial derivatives of the negative log-likelihood $L(D|\pi)$ are given by:

$$\frac{\partial L(D|\pi)}{\partial \pi_i} = \sum_{i \in W_i} \left( \frac{1}{\sum_{t \in A_i} \pi_t} - \frac{1}{\pi_i} \right) + \sum_{i \in L_i} \frac{1}{\sum_{t \in A_i} \pi_t},$$

\[ \text{(22)} \]

for all $i \in [n]$, where $W_i = \{\ell \mid i \in A_\ell, c_\ell = i\}$ is the set of observations where sample $i \in [n]$ is chosen and $L_i = \{\ell \mid i \in A_\ell, c_\ell \neq i\}$ is the set of observations where sample $i \in [n]$ is not chosen. Setting $\frac{\partial L(D|\pi)}{\partial \pi_i}$ from Eq. (22) to Eq. (21), we have:

$$\frac{\partial L_\rho(\pi, W^k, y^k)}{\partial \pi_i} = \sum_{i \in W_i} \left( \frac{1}{\sum_{t \in A_i} \pi_t} - \frac{1}{\pi_i} \right) + \sum_{i \in L_i} \frac{1}{\sum_{t \in A_i} \pi_t} + \sigma_i(\pi) = 0,$$

\[ \text{(23)} \]

for all $i \in [n]$. Multiplying both sides of Eq. (23) with $-\pi_i, i \in [n]$, we have:

$$\sum_{i \in W_i} \left( \frac{\sum_{j \neq i \in A_i} \pi_j}{\sum_{t \in A_i} \pi_t} \right) - \sum_{i \in L_i} \left( \frac{\pi_i}{\sum_{t \in A_i} \pi_t} \right) = -\pi_i \sigma_i(\pi) = 0,$$

\[ \text{(24)} \]

for all $i \in [n]$. Note that $\sum_{i \in W_i} \sum_{j \neq i \in A_i} = \sum_{i \neq j \in W_i \cap L_j} \cdot$ and $\sum_{i \in L_i} \cdot = \sum_{j \neq i \in W_j \cap L_i} \cdot$. Accordingly, we rewrite Eq. (24) as:

$$\sum_{j \neq i \in W_i \cap L_j} \left( \frac{\pi_j}{\sum_{t \in A_i} \pi_t} \right) - \sum_{j \neq i \in W_i \cap L_i} \left( \frac{\pi_i}{\sum_{t \in A_i} \pi_t} \right) = -\pi_i \sigma_i(\pi) = 0,$$

\[ \text{(25)} \]

for all $i \in [n]$. Then, the stationarity condition given by Eq. (20a) is equivalent to:

$$\sum_{j \neq i} \pi_j \lambda_{ji}(\pi) = \sum_{j \neq i} \pi_i \lambda_{ij}(\pi) = \pi_i \sigma_i(\pi) \quad \forall i \in [n],$$

\[ \text{(26)} \]

where $\lambda_{ji}(\pi), i, j \in [n], i \neq j$ are given by Eq. (12).

It is not evident that Eq. (26) corresponds to the balance equations of an MC as, in general, $\sigma(\pi) = [\sigma_i(\pi)]_{i \in [n]} \neq 0$. Nevertheless, for $\sigma_i(\pi) = \rho \frac{\partial D_\rho(\pi||\bar{\pi})}{\partial \pi_i} + y^k_i$, $i \in [n]$, Eq. (26) has the same form as the balance equations in Theorem 4.2 established by Yıldız et al. (2020). By this theorem, a stationary $\pi \in \mathbb{R}_+^n$ satisfying (20a) is also the stationary distribution of the continuous-time MC with transition rates given by Eq. (10).

B Datasets

Retinopathy of Prematurity (ROP). The Retinopathy of Prematurity (ROP) dataset contains $n = 100$ vessel-segmented retina images with dimensions $d = 224 \times 224$ (Ataer-Cansizoglu, 2015). Experts are provided
with two images and are asked to choose the image with higher severity of the ROP disease. Five experts independently label 5941 image pairs; the resulting dataset contains $m = 29705$ pairwise comparisons. Note that some pairs are labelled more than once by different experts.

**International Conference on Learning Representations (ICLR).** The ICLR Dataset contains abstracts and reviewer ratings of 2561 papers that are submitted to ICLR 2020 conference and are available on OpenReview website (Sun, 2020). We choose the top $n = 100$ papers, and extract $d = 768$ numerical features from each abstract using the Deep Bidirectional Transformers (BERT) (Devlin et al., 2019) architecture, pre-trained on the Books Corpus dataset (Zhu et al., 2015) and English Wikipedia. We normalize $X$ to have 0 mean and unit variance over samples $[n]$. We generate all possible $m = 120,324(2, 248, 524) \ K = 3(4)$-way rankings w.r.t. the relative order of the average reviewer ratings. We add noise to the resulting rankings following the same process as Movehub-Cost.

**Movehub-Cost.** The Movehub-Cost dataset contains the total ranking of 216 cities w.r.t. cost of living (Blitzer, 2017). Each city is associated with $d = 6$ numerical features, which are average costs for cappuccino, cinema, wine, gasoline, rent, and disposable income. We normalize $X$ to have 0 mean and unit variance over samples $[n]$. We select $n = 50$ cities and generate all possible $m = 230,298(2, 118, 756) \ K = 4(5)$-way rankings w.r.t. the relative order of the queried cities in the total ranking. To mimic the real-life noise introduced by human labelling, we apply the following post-processing to the resulting rankings: For each ranking, we sample a value uniformly at random in $[0, 1]$. If the value is less than 0.1, we add noise to the ranking by a cyclic permutation of the ranked samples.

**Movehub-Quality.** The Movehub-Quality dataset contains total ranking of the same 216 cities as Movehub-Cost, this time w.r.t. quality of life. Each city is associated with $d = 5$ numerical features, including overall scores for purchase power, healthcare, pollution, quality of life, and crime. We normalize $X$ to have 0 mean and unit variance over samples $[n]$. We select $n = 50$ cities and generate all possible $m = 230,298(2, 118, 756) \ K = 4(5)$-way rankings w.r.t. the relative order of the queried cities in the total ranking. We add noise to the rankings following the same process as Movehub-Cost.

**IMDB.** The IMDB Movies Dataset contains IMDB ratings of 14,762 movies, each of which is associated with $d = 36$ numerical features (Leka, 2016). We normalize $X$ to have 0 mean and unit variance over samples $[n]$. We select $n = 50$ movies and generate all possible $m = 85,583 \ K = 4$-way rankings w.r.t. the relative order of the ratings of queried movies. We add noise to the resulting rankings following the same process as Movehub-Cost.