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Survival of the Strictest: Stable and Unstable Equilibria under Regularized Learning with Partial Information

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Abstract

In this paper, we examine the Nash equilibrium convergence properties of no-regret learning in general N-player games. For concreteness, we focus on the archetypal *"follow the regularized leader"* (FTRL) family of algorithms, and we consider the full spectrum of uncertainty that the players may encounter – from noisy, oracle-based feedback, to bandit, payoff-based information. In this general context, we establish a comprehensive equivalence between the stability of a Nash equilibrium and its support: *a Nash equilibrium is stable and attracting with arbitrarily high probability if and only if it is strict* (i.e., each equilibrium strategy has a unique best response). This equivalence extends existing continuous-time versions of the "folk theorem" of evolutionary game theory to a bona fide algorithmic learning setting, and it provides a clear refinement criterion for the prediction of the day-to-day behavior of no-regret learning in games.

In more detail, we address the following questions: *Is there a class of Nash equilibria that consistently attract no-regret processes?* Conversely, *are all Nash equilibria equally likely to emerge as outcomes of a no-regret learning process?* To address them in a general setting, we focus on the *"follow the regularized leader"* (FTRL) algorithm and we prove the following result

$$x^* \text{ is a strict Nash equilibrium} \iff x^* \text{ is stochastically asymptotically stable under FTRL}$$

Formally, we get the following precise statements for a range of specific feedback models:

Theorem 1. *Let $x^* \in \mathcal{X}$ be a strict Nash equilibrium of the game under study. If FTRL is run with inexact payoff vector estimates with vanishing bias and moderately increasing variance, x^* is stochastically asymptotically stable.*

Theorem 2. *Let x^* be a mixed Nash equilibrium of a generic game. If FTRL is run with inexact payoff vector estimates with vanishing bias and moderately increasing variance, x^* is not stochastically asymptotically stable.*

These results – and, in particular, the implications for the bandit case – provide a learning justification to the abundance of arguments that have been made in the refinement literature against selecting mixed Nash equilibria [3, 6] and strengthen existing results on continuous-time game dynamics [1, 2, 5], sometimes referred to as the "folk theorem" of evolutionary game theory [4].

Keywords: No-regret learning, Nash Equilibrium, follow the regularized leader, asymptotic stability.

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