It was “all” for “nothing”:
sharp phase transitions for noiseless discrete channels

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Abstract
We prove a phase transition known as the “all-or-nothing” phenomenon for noiseless discrete channels. This class of models includes the Bernoulli group testing model and the planted Gaussian perceptron model. Previously, the existence of the all-or-nothing phenomenon for such models was only known in a limited range of parameters. Our work extends the results to all signals with sublinear sparsity.

Keywords: Phase transitions, group testing, teacher-student, perceptron, all-or-nothing

1. Extended Abstract
A surprising feature of high-dimensional inference problems is the presence of phase transitions (see, e.g. Mezard and Montanari, 2009). A particularly striking phase transition is known as the all-or-nothing phenomenon (Gamarnik and Zadik, 2017; Niles-Weed and Zadik, 2020; Reeves et al., 2019; Barbier et al., 2020; Luneau et al., 2020; Truong et al., 2020; Reeves et al., 2019; Zadik, 2019). In problems evincing this phenomenon, there is a sharp break: for some critical number of samples $n^*$, with $(1 - \epsilon)n^*$ samples it is impossible to infer almost any information about a parameter of interest, but with $(1 + \epsilon)n^*$ samples it is possible to infer the parameter almost perfectly.

In this work, we establish the all-or-nothing phenomenon for a general class of models we call “noiseless discrete channels.” We highlight two important special cases where our results significantly generalize previous work.

- **Bernoulli Group testing** (Dorfman, 1943): Assume a population of $N$ individuals, on which we perform group tests to learn the identities of $k = o(N)$ infected individuals. Each individual participates in each test independently w.p. $\nu/k$, and the test outcome is whether some group member is infected or not. Assuming $q = (1 - \nu/k)^k \leq 1/2$, we establish the all-or-nothing phenomenon for this task takes place at $n^* = \lceil k \log \frac{N}{k} / h(q) \rceil$ tests. This generalizes the work of Truong et al. (2020) which proves the result when $\log k = o(\log N)$ and $q = 1/2$.

- **Planted Gaussian perceptron** (Zdeborová and Krzakala, 2016): Here we assume hidden $k$-sparse weights $\theta \in \{0, 1\}^N$ and we observe i.i.d. samples of the form $(X_i, \mathbb{1}(\langle X_i, \theta \rangle \geq 0))$, where $X_i \sim N(0, I_N)$. Assuming $k = o(N)$, we establishing the all-or-nothing phenomenon takes places at $n^* = \lceil k \log \frac{N}{k} \rceil$ samples. Our result generalizes the work of (Luneau et al., 2020) which proves the result when $\omega(N^{5/9}) = k = o(N)$.

References


