APPENDIX—SUPPLEMENTARY MATERIAL

Kernelization.

In this appendix, we explicitly derive the kernelization of the approach proposed in Section 2.3 to learn a function f which is a non-linear function of the inputs.

Assuming that each object is described by a vector $\phi_x(o)$ lying in the feature space Φ corresponding to some kernel k, then the transformed vectors $x'(S')$ defined in Eqn. (10) become vectors $\phi'_x(S')$ from Φ computed by:

$$\phi'_x(S') = \frac{1}{N_+}\sum_{o\in S'}\phi_x(o) - \frac{1}{N_+}\sum_{o\in S_+}\phi_x(o),$$

and the dot-products $k'(S'_i, S'_j) = \phi'_x(S'_i)^T\phi'_x(S'_j)$ between two such vectors is given by:

$$\frac{1}{N_+^2}(\sum_{o_1\in S'_1, o_2\in S'_2} k(o_1,o_2) - \sum_{o_1\in S'_1, o_+\in S'_2} k(o_1,o_+)$$
$$- \sum_{o_2\in S'_2, o_+\in S'_2} k(o_2,o_+) + \sum_{o_{+,1}\in S_+, o_{+,2}\in S_+} k(o_{+,1},o_{+,2})),$$

which can be computed from the sole knowledge of the kernel k. We can thus directly use the kernel formulation of one-class SVM to learn a vector w of the following form:

$$w = \sum_{i=1}^{T}\alpha_i\phi'_x(S'_i).$$

To make predictions, objects in S can then be ranked according to:

$$f(\phi_x(o)) = \sum_{i=1}^{T}\alpha_i\phi'_x(S'_i)^T\phi_x(o),$$

which may be written in terms of the kernel k as follows:

$$f(\phi_x(o)) = \sum_{i=1}^{T}\alpha_i\frac{1}{N_+}\sum_{o'\in S'_i}k(o',o) - \frac{1}{N_+}\sum_{o'\in S_+}k(o',o),$$

making use of the fact that $\sum_i\alpha_i = 1$.