A MILPs for Bayesian Decisions

Stochastic programming (e.g. [1]) is a way to incorporate a probability distribution over future events directly into a mathematical program, in our case a MILP. The objective then becomes to maximize the *expected* utility subject to the constraints of the MILP. Here we give a small example to illustrate how such a MILP efficiently implements Bayesian decision theory.

Consider a domain with three agents and one task, where at planning we know that with probability 0.6 the task will require agent 1 and agent 2 (scenario 1), and with probability 0.4 it will require agent 3 (scenario 2). Our job is to select the team of agents most likely to accomplish the task.

We further impose a resource constraint: we have 4 units of fuel. If chosen, agent 1 uses 2 units of fuel, agent 2 uses 1 unit, and agent 3 uses 4 units.

Now it is straightforward to formulate the stochastic MILP. Let $x_i \in \{0, 1\}$ denote whether agent i is chosen by the plan, and $z_j \in \{0, 1\}$ whether the plan accomplishes the task in scenario j. Then we can write,

$$\begin{aligned} \max\ 0.6z_1 + 0.4z_2 \\ z_1 \leq x_1, z_1 \leq x_2 \\ z_2 \leq x_3 \\ 2x_1 + x_2 + 4x_3 \leq 4. \end{aligned}$$

Solving the MILP gives us the plan that chooses agent 1 and 2, with probability of success of 0.6. (Note, however, that solving the LP relaxation chooses agents 1 and 2, and 1/4 of agent 3, which has a higher probability of success, 0.7, but is infeasible.)

While this example is simple, it demonstrates how we might combine this paper's algorithm with reasoning about uncertain outcomes: for example, each individual agent's decision problem can contain randomness as illustrated above. More interestingly, the individual problems could contain multiple stages of decisions interleaved with random outcomes, as in a stochastic program with recourse. Finally, we can even add randomness at the inter-agent level: e.g., the number of units of a resource that are available could be a random variable, revealed to the agents at any point during the plan, or with information provided incrementally throughout the plan.

References

[1] W. B. Powell. A stochastic formulation of the dynamic assignment problem, with an application to truckload motor carriers. *Transportation Sci.*, 30(3):195–219, 1996.