

Supplementary Material

Proof of Lemma 4

The proof uses the following result of (Scott, 2012):

Lemma 8 ((Scott, 2012)). *Let $c \in (0, 1)$. For any $h : \mathcal{X} \rightarrow \{\pm 1\}$,*

$$\text{regret}_D^{0-1,(c)}[h] = \mathbf{E}_x [|\eta(x) - c| \cdot \mathbf{1}(h(x) \neq \text{sign}(\eta(x) - c))].$$

Proof of Lemma 4. The proof follows from Lemma 8 by a straightforward application of Jensen's inequality. In particular, we have,

$$\begin{aligned} & \text{regret}_D^{0-1,(c)}[h] \\ & \leq \mathbf{E}_x \left[|\eta(x) - c| \cdot \mathbf{1}((\hat{\eta}(x) - c)(\eta(x) - c)) \leq 0 \right], \\ & \quad \text{by Lemma 8 and definition of } h \\ & \leq \mathbf{E}_x \left[|\hat{\eta}(x) - \eta(x)| \right], \\ & \quad \text{since } (\hat{\eta}(x) - c)(\eta(x) - c) \leq 0 \implies \\ & \quad \quad |\eta(x) - c| \leq |\hat{\eta}(x) - \eta(x)| \\ & = \left(\left(\mathbf{E}_x \left[|\hat{\eta}(x) - \eta(x)| \right] \right)^r \right)^{1/r} \\ & \leq \left(\mathbf{E}_x \left[|\hat{\eta}(x) - \eta(x)|^r \right] \right)^{1/r}, \end{aligned}$$

where the last inequality follows by convexity of the function $g(z) = z^r$ ($r \geq 1$) and Jensen's inequality. \square

Proof of Theorem 5

Proof. Let $\gamma : \{\pm 1\} \times [0, 1] \rightarrow \bar{\mathbb{R}}_+$ be an underlying strongly proper loss such that

$$\ell(y, f) = \gamma(y, \psi^{-1}(f)).$$

Then we have,

$$\begin{aligned} & \mathbf{E}_x \left[(\hat{\eta}_S(x) - \eta(x))^2 \right] \\ & = \mathbf{E}_x \left[(\psi^{-1}(f_S(x)) - \eta(x))^2 \right], \\ & \quad \text{by definition of } \hat{\eta}_S \text{ (see Algorithm 1)} \\ & \leq \frac{1}{\kappa} \mathbf{E}_x \left[L_\gamma(\eta(x), \psi^{-1}(f_S(x))) - H_\gamma(\eta(x)) \right], \\ & \quad \text{for some } \kappa > 0, \text{ by strong properness of } \gamma \\ & = \frac{1}{\kappa} \mathbf{E}_x \left[L_\ell(\eta(x), f_S(x)) - H_\ell(\eta(x)) \right], \\ & \quad \text{since } L_\ell(\eta, f) = L_\gamma(\eta, \psi^{-1}(f)) \text{ and} \\ & \quad \text{since } H_\ell(\eta) = H_\gamma(\eta) \text{ (easy to verify)} \\ & = \frac{1}{\kappa} \text{regret}_D^\ell[f_S]. \end{aligned}$$

Thus, if K and λ_n can be chosen such that $\text{regret}_D^\ell[f_S] \xrightarrow{P} 0$, then $\mathbf{E}_x [(\hat{\eta}_S(x) - \eta(x))^2] \xrightarrow{P} 0$. The result then follows from Theorem 3. \square

Proof of Lemma 7

We will need the following results of (Scott, 2012):

Lemma 9 ((Scott, 2012)). *Let $\ell : \{\pm 1\} \times \bar{\mathbb{R}} \rightarrow \bar{\mathbb{R}}_+$ be any loss and let $c \in (0, 1)$. Let $w_c : [0, 1] \rightarrow (0, 1)$ be defined as*

$$w_c(\eta) = \eta(1 - c) + (1 - \eta)c.$$

Then for any $\eta \in [0, 1]$ and $f \in \bar{\mathbb{R}}$,

$$L_\ell^{(c)}(\eta, f) = w_c(\eta) \cdot L_\ell \left(\frac{\eta(1 - c)}{w_c(\eta)}, f \right).$$

Lemma 10 ((Scott, 2012)). *Let $\ell : \{\pm 1\} \times \bar{\mathbb{R}} \rightarrow \bar{\mathbb{R}}_+$ be classification-calibrated at $\frac{1}{2}$ and let $c \in (0, 1)$. Then $\ell^{(c)}$ is classification-calibrated at c .*

Proof of Lemma 7. We first show that under the given condition on ℓ , we have $\forall \eta \in [0, 1]$,

$$L_\ell^{(c)}(\eta, 0) - H_\ell^{(c)}(\eta) \geq \frac{\alpha}{2^r} |\eta - c|^r. \quad (3)$$

To see this, note that for any $\eta \in [0, 1]$,

$$\begin{aligned} & L_\ell^{(c)}(\eta, 0) - H_\ell^{(c)}(\eta) \\ & = w_c(\eta) \cdot \left(L_\ell \left(\frac{\eta(1 - c)}{w_c(\eta)}, 0 \right) - H_\ell \left(\frac{\eta(1 - c)}{w_c(\eta)} \right) \right), \\ & \quad \text{by Lemma 9} \\ & \geq w_c(\eta) \cdot \alpha \left| \frac{\eta(1 - c)}{w_c(\eta)} - \frac{1}{2} \right|^r, \quad \text{by assumption} \\ & = w_c(\eta) \cdot \alpha \left| \frac{\eta - c}{2w_c(\eta)} \right|^r \\ & = \frac{\alpha}{2^r} \frac{|\eta - c|^r}{(w_c(\eta))^{r-1}} \\ & \geq \frac{\alpha}{2^r} |\eta - c|^r, \end{aligned}$$

where $w_c(\eta)$ is as defined in Lemma 9 and the last inequality follows since $w_c(\eta) = \eta(1 - c) + (1 - \eta)c \leq (1 - c) + c = 1$ and $r - 1 \geq 0$. This proves the claim in Eq. (3).

Next, we claim that for all $\eta \in [0, 1]$ and $f \in \bar{\mathbb{R}}$,

$$f(\eta - c) \leq 0 \implies L_\ell^{(c)}(\eta, 0) \leq L_\ell^{(c)}(\eta, f). \quad (4)$$

To see this, let $\eta \in [0, 1]$, and let $f \in \bar{\mathbb{R}}$ be such that $f(\eta - c) \leq 0$. Let f^* denote any minimizer of $L_\ell^c(\eta, f)$. Since ℓ is classification-calibrated at $\frac{1}{2}$, we have by Theorem 10 that $\ell^{(c)}$ is classification-calibrated at c , and therefore $f^*(\eta - c) > 0$. Moreover, since ℓ is convex in its second argument, we have that $\ell^{(c)}$ is convex in its second argument, which in turn implies $L_\ell^{(c)}$ is convex in its second argument. We consider the following three cases:

1. $\eta > c$: Then $f \leq 0$ and $f^* > 0$. Therefore, by convexity of $L_\ell^{(c)}$ in its second argument, we have

$$\begin{aligned} L_\ell^{(c)}(\eta, 0) &\leq \max(L_\ell^{(c)}(\eta, f), L_\ell^{(c)}(\eta, f^*)) \\ &= L_\ell^{(c)}(\eta, f). \end{aligned}$$

2. $\eta < c$: Then $f \geq 0$ and $f^* < 0$. Again, by convexity of $L_\ell^{(c)}$ in its second argument, we have

$$\begin{aligned} L_\ell^{(c)}(\eta, 0) &\leq \max(L_\ell^{(c)}(\eta, f), L_\ell^{(c)}(\eta, f^*)) \\ &= L_\ell^{(c)}(\eta, f). \end{aligned}$$

3. $\eta = c$: In this case, $w_c(\eta) = 2c(1 - c)$, and so

$$L_\ell^{(c)}(\eta, f) = 2c(1 - c) \cdot L_\ell\left(\frac{1}{2}, f\right),$$

which is minimized by $f^* = 0$ (by the assumption that $L_\ell\left(\frac{1}{2}, 0\right) = H_\ell\left(\frac{1}{2}\right)$). This gives

$$L_\ell^{(c)}(\eta, 0) = L_\ell^{(c)}(\eta, f^*) \leq L_\ell^{(c)}(\eta, f).$$

This proves the claim in Eq. (4).

Putting everything together, we have for any $f : \mathcal{X} \rightarrow \bar{\mathbb{R}}$ and $h(x) = \text{sign}(f(x))$:

$$\begin{aligned} \text{regret}_D^{0-1,(c)}[h] &\leq \mathbf{E}_x \left[|\eta(x) - c| \cdot \mathbf{1}(f(x)(\eta(x) - c) \leq 0) \right], \\ &\quad \text{by Lemma 8 and definition of } h \\ &= \left(\left(\mathbf{E}_x \left[|\eta(x) - c| \cdot \mathbf{1}(f(x)(\eta(x) - c) \leq 0) \right] \right)^r \right)^{1/r} \\ &\leq \left(\mathbf{E}_x \left[|\eta(x) - c|^r \cdot \mathbf{1}(f(x)(\eta(x) - c) \leq 0) \right] \right)^{1/r}, \\ &\quad \text{by convexity of the function } g(z) = z^r \ (r \geq 1) \\ &\quad \text{and Jensen's inequality} \\ &\leq \frac{1}{\alpha^{1/r}} \left(\mathbf{E}_x \left[\left(L_\ell^{(c)}(\eta(x), 0) - H_\ell^{(c)}(\eta(x)) \right) \cdot \mathbf{1}(f(x)(\eta(x) - c) \leq 0) \right] \right)^{1/r}, \\ &\quad \text{by Eq. (3)} \\ &\leq \frac{1}{\alpha^{1/r}} \left(\mathbf{E}_x \left[\left(L_\ell^{(c)}(\eta(x), f(x)) - H_\ell^{(c)}(\eta(x)) \right) \right] \right)^{1/r}, \\ &\quad \text{by Eq. (4)} \\ &= \frac{1}{\alpha^{1/r}} \left(\text{regret}_D^{\ell,(c)}[f] \right)^{1/r}. \end{aligned}$$

This proves the lemma. \square

Proof of Theorem 6

Proof. By Lemma 7, we have

$$\text{regret}_D^{0-1,(\hat{p}_S)}[h_S] \leq \frac{2}{\alpha^{1/r}} \left(\text{regret}_D^{\ell,(\hat{p}_S)}[f_S] \right)^{1/r}.$$

Thus, if K and λ_n can be chosen such that $\text{regret}_D^{\ell,(\hat{p}_S)}[f_S] \xrightarrow{P} 0$, then $\text{regret}_D^{0-1,(\hat{p}_S)}[h_S] \xrightarrow{P} 0$. The result then follows from Lemma 2. \square

Calculation of AM-Regret for Synthetic Data

Let D be a probability distribution on $\mathbb{R}^d \times \{\pm 1\}$, with $\mathbf{P}(y = 1) = p$ for some $p \in (0, 1)$, and under which positive instances are drawn according to a multivariate Gaussian distribution $\mathcal{N}(\mu, \Sigma)$ and negative instances are drawn according to $\mathcal{N}(-\mu, \Sigma)$, where $\Sigma \in \mathbb{R}^{d \times d}$ is a symmetric positive definite matrix and $\mu \in \mathbb{R}^d$.

Then the AM measure of a classifier $h : \mathbb{R}^d \rightarrow \{\pm 1\}$ w.r.t. D is given by

$$\text{AM}_D[h] = \frac{\text{TPR}_D[h] + \text{TNR}_D[h]}{2}.$$

For any linear classifier $h(x) = \text{sign}(w^\top x + b)$, where $w \in \mathbb{R}^d$ and $b \in \mathbb{R}$, this can be written as

$$\begin{aligned} \text{AM}_D[h] &= \frac{1}{2} \left[\int_{\{x | w^\top x > b\}} f_+(x) \, dx \right. \\ &\quad \left. + \int_{\{x | w^\top x \leq b\}} f_-(x) \, dx \right], \end{aligned}$$

where f_+ is the pdf corresponding to the distribution $\mathcal{N}(\mu, \Sigma)$ and f_- is the pdf corresponding to $\mathcal{N}(-\mu, \Sigma)$. This in turn can be converted to an expression involving one-dimensional integrations:

$$\text{AM}_D[h] = \frac{1}{2} \left[\int_b^\infty g_+(z) \, dz + \int_{-\infty}^b g_-(z) \, dz \right],$$

where g_+ is the pdf corresponding to the distribution $\mathcal{N}(w^\top \mu, w^\top \Sigma w)$ and g_- is the pdf corresponding to $\mathcal{N}(-w^\top \mu, w^\top \Sigma w)$.

Moreover, for a distribution of the above form, it can be verified that the optimal classifier with respect to the AM measure, $h_p^* : \mathbb{R}^d \rightarrow \{\pm 1\}$ given by $h_p^*(x) = \text{sign}(\eta(x) - p)$ where $\eta(x) = \mathbf{P}(y = 1|x)$, is a linear classifier. In particular, from standard results, we have

$$\eta(x) = \frac{pf_+(x)}{pf_+(x) + (1-p)f_-(x)} = \frac{1}{1 + e^{-f(x)}},$$

where

$$f(x) = \ln \left(\frac{pf_+(x)}{(1-p)f_-(x)} \right)$$

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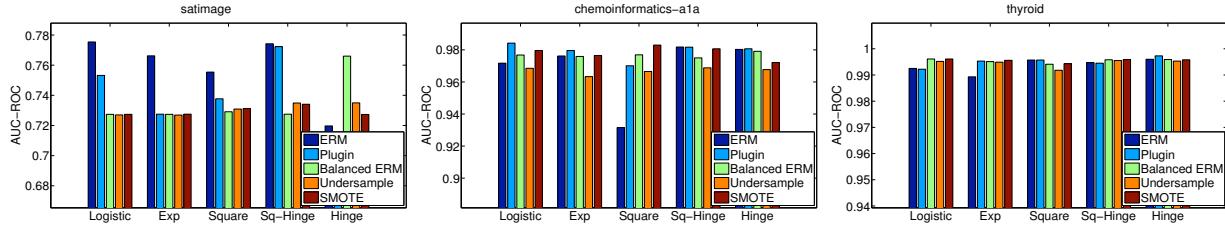


Figure 3. Results on the three data sets summarized in Table 3, using various algorithms in conjunction with different loss functions, in terms of AUC-ROC; higher values are better (see Section 6.2).

$$\begin{aligned}
&= -\frac{1}{2}(x - \mu)^\top \Sigma^{-1}(x - \mu) \\
&\quad + \frac{1}{2}(x + \mu)^\top \Sigma^{-1}(x + \mu) + \ln \left(\frac{p}{1-p} \right) \\
&= 2\mu^\top \Sigma^{-1}x + \ln \left(\frac{p}{1-p} \right).
\end{aligned}$$

This can be seen to yield

$$\begin{aligned}
h_p^*(x) &= \text{sign} \left(\frac{1}{1 + e^{-f(x)}} - p \right) \\
&= \text{sign} \left(f(x) - \ln \left(\frac{p}{1-p} \right) \right) \\
&= \text{sign} (2\mu^\top \Sigma^{-1}x) \\
&= \text{sign}(w^\top x), \quad \text{where } w = \Sigma^{-1}\mu.
\end{aligned}$$

Thus the optimal AM value, $\text{AM}_D^* = \text{AM}_D[h_p^*]$, can be computed similarly as for any other linear classifier above, using $w = \Sigma^{-1}\mu$ and $b = 0$. The AM-regret of any linear classifier h w.r.t. D can then be computed as

$$\text{regret}_D^{\text{AM}}[h] = \text{AM}_D[h_p^*] - \text{AM}_D[h].$$

Additional Results on Real Data

Table 4 summarizes all 17 real data sets used in our experiments described in Section 6.2. Tables 5, 6, and 7 give results with all algorithms on all data sets, in terms of the AM, GM, and AUC-ROC performance measures, respectively. Tables 8, 9, and 10 give the average ranks of different algorithms (for each loss function) over the 17 data sets, based on mean AM, GM, and AUC-ROC performance, respectively. The AUC-ROC performance of all algorithms on the 3 data sets discussed in detail in Section 6.2 is also shown in the form of plots in Figure 3.

Table 4. Summary of 17 real data sets.

Data set	# examples	# features	$p = P(y = 1)$
abalone	4177	8	0.0077
car	1728	8	0.0376
chemo-a1a	2142	1021	0.0233
chemo-pde5	2142	1021	0.0233
covtype-binary	38501	54	0.0713
german	1000	24	0.3000
kddcup08	102294	117	0.0061
kddcup98	191779	15	0.0507
letter	20000	16	0.0367
optdigits	5620	64	0.0986
pendigits	10992	16	0.0960
satimage	6435	36	0.0973
segment	2310	19	0.1429
shuttle	58000	9	0.0004
spambase	4601	57	0.3940
splice	3190	61	0.2404
thyroid	7200	21	0.0231

Table 8. Average ranks for all algorithms (for each loss function) based on mean AM performance. Lower ranks are better.

Algorithm	Logistic	Exp	Square	Sq-Hinge	Hinge
ERM	4.78	4.68	4.42	4.63	4.47
Plugin	2.94	1.89	3.78	2.94	2.68
Balanced ERM	1.94	2.36	1.89	1.94	2.10
Undersample	2.94	3.31	2.68	2.84	3.26
SMOTE	2.21	2.52	1.84	2.36	2.42

Table 9. Average ranks for all algorithms (for each loss function) based on mean GM performance. Lower ranks are better.

Algorithm	Logistic	Exp	Square	Sq-Hinge	Hinge
ERM	4.78	4.68	4.47	4.68	4.52
Plugin	3.0	1.89	3.73	3.0	2.63
Balanced ERM	1.89	2.42	1.89	2.0	2.31
Undersample	2.94	3.21	2.68	2.89	3.10
SMOTE	2.15	2.57	1.84	2.31	2.36

Table 10. Average ranks for all algorithms (for each loss function) based on mean AUC-ROC performance. Lower ranks are better.

Algorithm	Logistic	Exp	Square	Sq-Hinge	Hinge
ERM	2.68	2.31	3.47	2.78	3.21
Plugin	2.52	1.68	3.15	3.10	2.73
Balanced ERM	2.15	2.36	2.15	2.26	2.42
Undersample	4.05	4.78	3.42	3.73	3.63
SMOTE	2.42	2.63	2.36	2.42	2.68

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Table 5. Results for all algorithms on all data sets, in terms of AM. For each loss function, the ranks of different algorithms are displayed in parentheses (i.e. ranks are column-relative within each data set block). Higher AM values are better; lower ranks are better.

Dataset	Algorithm	Logistic	Exp	Square	Sq-Hinge	Hinge
abalone	ERM	0.4999 ± 0.0003 (5)	0.4996 ± 0.0004 (5)	0.5000 ± 0.0000 (5)	0.5000 ± 0.0000 (5)	0.5000 ± 0.0000 (5)
	Plugin	0.6846 ± 0.0804 (4)	0.7363 ± 0.0872 (1)	0.7618 ± 0.0594 (1)	0.7052 ± 0.0689 (4)	0.5348 ± 0.0688 (4)
	Balanced ERM	0.7570 ± 0.0701 (1)	0.7357 ± 0.0859 (2)	0.7295 ± 0.0851 (3)	0.7375 ± 0.0790 (1)	0.7694 ± 0.0618 (1)
	Undersample	0.7333 ± 0.0724 (2)	0.7117 ± 0.0856 (4)	0.7306 ± 0.0750 (2)	0.7284 ± 0.0821 (2)	0.7564 ± 0.0723 (2)
	SMOTE	0.7300 ± 0.0804 (3)	0.7185 ± 0.0704 (3)	0.7135 ± 0.0709 (4)	0.7140 ± 0.0904 (3)	0.7180 ± 0.0853 (3)
car	ERM	0.9285 ± 0.0496 (5)	0.9245 ± 0.0594 (5)	0.5000 ± 0.0000 (5)	0.9400 ± 0.0509 (5)	0.9481 ± 0.0641 (5)
	Plugin	0.9938 ± 0.0027 (3)	0.9947 ± 0.0028 (1)	0.7794 ± 0.0135 (4)	0.9299 ± 0.0044 (3)	0.9901 ± 0.0081 (3)
	Balanced ERM	0.9944 ± 0.0027 (1)	0.9947 ± 0.0028 (1)	0.9318 ± 0.0064 (2)	0.9940 ± 0.0033 (1)	0.9911 ± 0.0036 (2)
	Undersample	0.9711 ± 0.0116 (4)	0.9710 ± 0.0119 (4)	0.9243 ± 0.0131 (3)	0.9752 ± 0.0109 (4)	0.9856 ± 0.0144 (4)
	SMOTE	0.9944 ± 0.0026 (1)	0.9941 ± 0.0032 (3)	0.9326 ± 0.0062 (1)	0.9936 ± 0.0037 (2)	0.9937 ± 0.0038 (1)
chemo-a1a	ERM	0.8274 ± 0.1060 (5)	0.8464 ± 0.0887 (5)	0.8140 ± 0.0771 (4)	0.8511 ± 0.0754 (5)	0.8986 ± 0.0575 (3)
	Plugin	0.9155 ± 0.0558 (1)	0.9085 ± 0.0578 (1)	0.8108 ± 0.0312 (5)	0.9223 ± 0.0572 (1)	0.9019 ± 0.0573 (2)
	Balanced ERM	0.8987 ± 0.0651 (3)	0.9044 ± 0.0592 (2)	0.8937 ± 0.0607 (3)	0.8842 ± 0.0524 (4)	0.9416 ± 0.0314 (1)
	Undersample	0.9058 ± 0.0427 (2)	0.9008 ± 0.0475 (3)	0.9048 ± 0.0549 (1)	0.9096 ± 0.0388 (2)	0.8934 ± 0.0543 (4)
	SMOTE	0.8918 ± 0.0395 (4)	0.8765 ± 0.0425 (4)	0.9021 ± 0.0575 (2)	0.9089 ± 0.0526 (3)	0.8667 ± 0.0480 (5)
chemo-pde5	ERM	0.9164 ± 0.0689 (5)	0.9082 ± 0.0557 (5)	0.9064 ± 0.0775 (4)	0.9314 ± 0.0627 (4)	0.9348 ± 0.0621 (3)
	Plugin	0.9621 ± 0.0385 (1)	0.9351 ± 0.0609 (1)	0.8119 ± 0.0384 (5)	0.9633 ± 0.0334 (1)	0.9550 ± 0.0516 (1)
	Balanced ERM	0.9441 ± 0.0540 (2)	0.9315 ± 0.0654 (2)	0.9497 ± 0.0422 (2)	0.9416 ± 0.0580 (2)	0.9404 ± 0.0452 (2)
	Undersample	0.9285 ± 0.0374 (4)	0.9241 ± 0.0353 (4)	0.9271 ± 0.0349 (3)	0.9275 ± 0.0358 (5)	0.9283 ± 0.0413 (5)
	SMOTE	0.9408 ± 0.0597 (3)	0.9278 ± 0.0743 (3)	0.9516 ± 0.0392 (1)	0.9380 ± 0.0592 (3)	0.9318 ± 0.0718 (4)
covtype-binary	ERM	0.7157 ± 0.0113 (5)	0.7078 ± 0.0104 (5)	0.5882 ± 0.0058 (5)	0.7080 ± 0.0134 (5)	0.7232 ± 0.0104 (5)
	Plugin	0.8790 ± 0.0047 (4)	0.8840 ± 0.0048 (2)	0.8204 ± 0.0039 (4)	0.8752 ± 0.0054 (4)	0.8559 ± 0.0070 (4)
	Balanced ERM	0.8846 ± 0.0042 (3)	0.8842 ± 0.0050 (1)	0.8801 ± 0.0046 (2)	0.8845 ± 0.0041 (1)	0.8842 ± 0.0042 (3)
	Undersample	0.8853 ± 0.0046 (1)	0.8828 ± 0.0049 (4)	0.8799 ± 0.0047 (3)	0.8845 ± 0.0046 (1)	0.8848 ± 0.0044 (2)
	SMOTE	0.8851 ± 0.0038 (2)	0.8835 ± 0.0049 (3)	0.8807 ± 0.0049 (1)	0.8843 ± 0.0035 (3)	0.8859 ± 0.0042 (1)
german	ERM	0.6832 ± 0.0283 (5)	0.6788 ± 0.0239 (5)	0.6776 ± 0.0215 (5)	0.6839 ± 0.0225 (5)	0.6796 ± 0.0277 (5)
	Plugin	0.7118 ± 0.0266 (2)	0.7030 ± 0.0335 (4)	0.7107 ± 0.0295 (2)	0.7139 ± 0.0332 (2)	0.7191 ± 0.0257 (1)
	Balanced ERM	0.7079 ± 0.0337 (4)	0.7042 ± 0.0365 (3)	0.7058 ± 0.0347 (3)	0.7066 ± 0.0331 (4)	0.7059 ± 0.0370 (3)
	Undersample	0.7094 ± 0.0351 (3)	0.7151 ± 0.0290 (1)	0.7031 ± 0.0364 (4)	0.7070 ± 0.0362 (3)	0.7044 ± 0.0328 (4)
	SMOTE	0.7176 ± 0.0316 (1)	0.7080 ± 0.0313 (2)	0.7199 ± 0.0285 (1)	0.7193 ± 0.0287 (1)	0.7132 ± 0.0258 (2)
kddcup08	ERM	0.6326 ± 0.0162 (5)	0.6349 ± 0.0260 (5)	0.5004 ± 0.0012 (5)	0.5891 ± 0.0154 (5)	0.6224 ± 0.0150 (5)
	Plugin	0.8452 ± 0.0174 (1)	0.8294 ± 0.0226 (2)	0.7336 ± 0.0149 (4)	0.8396 ± 0.0182 (1)	0.8269 ± 0.0146 (1)
	Balanced ERM	0.8255 ± 0.0171 (3)	0.8259 ± 0.0149 (3)	0.8244 ± 0.0120 (2)	0.8288 ± 0.0179 (3)	0.8266 ± 0.0151 (2)
	Undersample	0.8056 ± 0.0183 (4)	0.8025 ± 0.0168 (4)	0.8120 ± 0.0162 (3)	0.8080 ± 0.0199 (4)	0.8030 ± 0.0197 (4)
	SMOTE	0.8296 ± 0.0169 (2)	0.8311 ± 0.0214 (1)	0.8272 ± 0.0139 (1)	0.8289 ± 0.0189 (2)	0.8243 ± 0.0188 (3)
kddcup98	ERM	0.5000 ± 0.0000 (5)	0.5000 ± 0.0000 (5)	0.5000 ± 0.0000 (5)	0.5000 ± 0.0000 (5)	0.5000 ± 0.0000 (4)
	Plugin	0.5861 ± 0.0054 (1)	0.5855 ± 0.0068 (1)	0.5845 ± 0.0055 (3)	0.5849 ± 0.0055 (2)	0.4897 ± 0.0216 (5)
	Balanced ERM	0.5853 ± 0.0060 (2)	0.5854 ± 0.0066 (2)	0.5853 ± 0.0063 (1)	0.5855 ± 0.0064 (1)	0.5816 ± 0.0063 (1)
	Undersample	0.5851 ± 0.0065 (3)	0.5848 ± 0.0062 (4)	0.5848 ± 0.0061 (2)	0.5848 ± 0.0061 (3)	0.5814 ± 0.0069 (2)
	SMOTE	0.5840 ± 0.0071 (4)	0.5849 ± 0.0065 (3)	0.5845 ± 0.0075 (3)	0.5839 ± 0.0071 (4)	0.5810 ± 0.0062 (3)
letter	ERM	0.8302 ± 0.0185 (5)	0.8196 ± 0.0192 (5)	0.5000 ± 0.0000 (5)	0.8281 ± 0.0162 (5)	0.8354 ± 0.0195 (5)
	Plugin	0.9386 ± 0.0149 (4)	0.9478 ± 0.0078 (3)	0.7590 ± 0.0048 (4)	0.9369 ± 0.0131 (4)	0.9229 ± 0.0174 (4)
	Balanced ERM	0.9527 ± 0.0059 (2)	0.9475 ± 0.0093 (4)	0.9387 ± 0.0059 (1)	0.9526 ± 0.0056 (2)	0.9540 ± 0.0060 (3)
	Undersample	0.9519 ± 0.0052 (3)	0.9484 ± 0.0071 (2)	0.9371 ± 0.0070 (3)	0.9524 ± 0.0051 (3)	0.9541 ± 0.0058 (2)
	SMOTE	0.9547 ± 0.0056 (1)	0.9494 ± 0.0076 (1)	0.9378 ± 0.0071 (2)	0.9536 ± 0.0060 (1)	0.9559 ± 0.0061 (1)
optdigits	ERM	0.9877 ± 0.0084 (5)	0.9872 ± 0.0080 (5)	0.9863 ± 0.0093 (4)	0.9864 ± 0.0075 (5)	0.9886 ± 0.0063 (4)
	Plugin	0.9893 ± 0.0055 (3)	0.9896 ± 0.0066 (1)	0.8722 ± 0.0041 (5)	0.9873 ± 0.0052 (4)	0.9908 ± 0.0064 (1)
	Balanced ERM	0.9900 ± 0.0066 (1)	0.9892 ± 0.0067 (2)	0.9925 ± 0.0035 (1)	0.9902 ± 0.0070 (1)	0.9895 ± 0.0073 (3)
	Undersample	0.9886 ± 0.0059 (4)	0.9880 ± 0.0077 (4)	0.9923 ± 0.0033 (3)	0.9888 ± 0.0065 (3)	0.9882 ± 0.0071 (5)
	SMOTE	0.9898 ± 0.0070 (2)	0.9884 ± 0.0071 (3)	0.9924 ± 0.0043 (2)	0.9899 ± 0.0065 (2)	0.9898 ± 0.0069 (2)
pendigits	ERM	0.9186 ± 0.0134 (5)	0.9047 ± 0.0152 (5)	0.6573 ± 0.0145 (5)	0.9160 ± 0.0152 (5)	0.9220 ± 0.0135 (5)
	Plugin	0.9556 ± 0.0075 (4)	0.9499 ± 0.0062 (3)	0.8032 ± 0.0072 (4)	0.9511 ± 0.0067 (4)	0.9508 ± 0.0095 (4)
	Balanced ERM	0.9597 ± 0.0049 (2)	0.9495 ± 0.0059 (4)	0.9336 ± 0.0043 (2)	0.9596 ± 0.0054 (1)	0.9602 ± 0.0052 (2)
	Undersample	0.9574 ± 0.0056 (3)	0.9518 ± 0.0072 (1)	0.9327 ± 0.0054 (3)	0.9583 ± 0.0055 (3)	0.9589 ± 0.0061 (3)
	SMOTE	0.9598 ± 0.0061 (1)	0.9515 ± 0.0043 (2)	0.9345 ± 0.0047 (1)	0.9591 ± 0.0064 (2)	0.9615 ± 0.0057 (1)
satimage	ERM	0.5111 ± 0.0040 (5)	0.5075 ± 0.0051 (5)	0.5000 ± 0.0000 (5)	0.5041 ± 0.0036 (5)	0.5000 ± 0.0000 (5)
	Plugin	0.7133 ± 0.0127 (4)	0.7321 ± 0.0137 (1)	0.7156 ± 0.0079 (4)	0.7275 ± 0.0089 (2)	0.6460 ± 0.0337 (4)
	Balanced ERM	0.7319 ± 0.0134 (1)	0.7320 ± 0.0136 (2)	0.7306 ± 0.0138 (1)	0.7312 ± 0.0140 (1)	0.7306 ± 0.0110 (1)
	Undersample	0.7288 ± 0.0188 (3)	0.7288 ± 0.0185 (4)	0.7272 ± 0.0172 (3)	0.7265 ± 0.0177 (3)	0.7223 ± 0.0128 (3)
	SMOTE	0.7312 ± 0.0135 (2)	0.7312 ± 0.0139 (3)	0.7281 ± 0.0132 (2)	0.7260 ± 0.0172 (4)	0.7261 ± 0.0126 (2)
segment	ERM	0.9962 ± 0.0049 (4)	0.9962 ± 0.0049 (2)	0.9970 ± 0.0039 (1)	0.9970 ± 0.0040 (2)	0.9984 ± 0.0030 (1)
	Plugin	0.9962 ± 0.0049 (4)	0.9961 ± 0.0048 (3)	0.9802 ± 0.0065 (5)	0.9975 ± 0.0032 (1)	0.9973 ± 0.0036 (3)
	Balanced ERM	0.9978 ± 0.0035 (1)	0.9961 ± 0.0052 (3)	0.9970 ± 0.0039 (1)	0.9970 ± 0.0040 (2)	0.9978 ± 0.0036 (2)
	Undersample	0.9969 ± 0.0042 (2)	0.9961 ± 0.0046 (3)	0.9970 ± 0.0038 (1)	0.9970 ± 0.0046 (2)	0.9957 ± 0.0041 (5)
	SMOTE	0.9969 ± 0.0039 (2)	0.9963 ± 0.0043 (1)	0.9970 ± 0.0038 (1)	0.9968 ± 0.0038 (5)	0.9971 ± 0.0043 (4)
shuttle	ERM	0.5410 ± 0.0451 (5)	0.5000 ± 0.0000 (5)	0.5000 ± 0.0000 (5)	0.5000 ± 0.0000 (5)	0.5300 ± 0.0483 (5)
	Plugin	0.6241 ± 0.0638 (2)	0.6388 ± 0.0648 (3)	0.5901 ± 0.0695 (4)	0.6146 ± 0.0559 (4)	0.6163 ± 0.0880 (3)
	Balanced ERM	0.6156 ± 0.0802 (3)	0.6389 ± 0.0649 (2)	0.6449 ± 0.0552 (2)	0.6379 ± 0.0650 (3)	0.6340 ± 0.0607 (2)
	Undersample	0.6548 ± 0.0825 (1)	0.6578 ± 0.0910 (1)	0.6474 ± 0.0939 (1)	0.6467 ± 0.0960 (1)	0.6602 ± 0.0913 (1)
	SMOTE	0.5954 ± 0.0694 (4)	0.6312 ± 0.0630 (4)	0.6384 ± 0.0539 (3)	0.6392 ± 0.0550 (2)	0.6156 ± 0.0998 (4)
spambase	ERM	0.9181 ± 0.0058 (5)	0.9081 ± 0.0094 (5)	0.8674 ± 0.0124 (5)	0.9171 ± 0.0069 (5)	0.9213 ± 0.0083 (5)
	Plugin	0.9247 ± 0.0066 (3)	0.9179 ± 0.0068 (2)	0.9034 ± 0.0086 (1)	0.9232 ± 0.0073 (3)	0.9269 ± 0.0082 (1)
	Balanced ERM	0.9252 ± 0.0065 (2)	0.9180 ± 0.0067 (1)	0.9010 ± 0.0073 (3)	0.9244 ± 0.0063 (2)	0.9233 ± 0.0078 (4)
	Undersample	0.9227 ± 0.0074 (4)	0.9153 ± 0.0074 (4)	0.8996 ± 0.0076 (4)	0.9214 ± 0.0074 (4)	0.9246 ± 0.0074 (3)
	SMOTE	0.9256 ± 0.0068 (1)	0.9174 ± 0.0067 (3)	0.9028 ± 0.0063 (2)	0.9252 ± 0.0063 (1)	0.9253 ± 0.0068 (2)
splice	ERM	0.9681 ± 0.0080 (2)	0.9666 ± 0.0062 (2)	0.9671 ± 0.0073 (1)	0.9701 ± 0.0073 (2)	0.9664 ± 0.0096 (5)
	Plugin	0.9630 ± 0.0088 (4)	0.9659 ± 0.0071 (3)	0.8907 ± 0.0104 (5)	0.9586 ± 0.0081 (5)	0.9702 ± 0.0072 (1)
	Balanced ERM	0.9664 ± 0.0076 (3)	0.9657 ± 0.0071 (4)	0.9600 ± 0.0070 (3)	0.9670 ± 0.0065 (3)	0.9686 ± 0.0078 (3)
	Undersample	0.9620 ± 0.0067 (5)	0.9612 ± 0.0067 (5)	0.9584 ± 0.0054 (4)	0.9640 ± 0.0057 (4)	0.9677 ± 0.0086 (4)
	SMOTE	0.9691 ± 0.0060 (1)	0.9674 ± 0.0069 (1)	0.9615 ± 0.0063 (2)	0.9706 ± 0.0064 (1)	0.9700 ± 0.0079 (2)
thyroid	ERM	0.8491 ± 0.0326 (5)	0.8497 ± 0.0323 (5)	0.6237 ± 0.0429 (5)	0.8515 ± 0.0300 (5)	0.8518 ± 0.0303 (5)
	Plugin	0.9669 ± 0.0144 (4)	0.9762 ± 0.0172 (1)	0.8302 ± 0.0062 (4)	0.9727 ± 0.0134 (4)	0.9838 ± 0.0045 (1)
	Balanced ERM	0.9766 ± 0.0186 (1)	0.9746 ± 0.0183 (2)	0.9747 ± 0.0078 (1)	0.9741 ± 0.0144 (2)	0.9759 ± 0.0188 (2)
	Undersample	0.9721 ± 0.0138 (2)	0.9717 ± 0.0162 (3)	0.9598 ± 0.0143 (3)	0.9743 ± 0.0155	

Statistical Consistency of Class Imbalance Methods

Table 6. Results for all algorithms on all data sets, in terms of GM. For each loss function, the ranks of different algorithms are displayed in parentheses (i.e. ranks are column-relative within each data set block). Higher GM values are better; lower ranks are better.

Dataset	Algorithm	Logistic	Exp	Square	Sq-Hinge	Hinge
abalone	ERM	0.0000 ± 0.0000 (5)	0.0000 ± 0.0000 (5)	0.0000 ± 0.0000 (5)	0.0000 ± 0.0000 (5)	0.0000 ± 0.0000 (5)
	Plugin	0.6761 ± 0.0787 (4)	0.7286 ± 0.0926 (1)	0.7405 ± 0.0501 (1)	0.6972 ± 0.0598 (4)	0.4238 ± 0.2115 (4)
	Balanced ERM	0.7528 ± 0.0707 (1)	0.7281 ± 0.0913 (2)	0.7216 ± 0.0906 (3)	0.7307 ± 0.0831 (1)	0.7661 ± 0.0604 (1)
	Undersample	0.7268 ± 0.0739 (2)	0.7039 ± 0.0893 (4)	0.7236 ± 0.0772 (2)	0.7216 ± 0.0843 (2)	0.7502 ± 0.0731 (2)
	SMOTE	0.7207 ± 0.0886 (3)	0.7115 ± 0.0757 (3)	0.7045 ± 0.0777 (4)	0.7015 ± 0.1017 (3)	0.7053 ± 0.0982 (3)
car	ERM	0.9247 ± 0.0542 (5)	0.9196 ± 0.0671 (5)	0.0000 ± 0.0000 (5)	0.9369 ± 0.0555 (5)	0.9444 ± 0.0723 (5)
	Plugin	0.9938 ± 0.0027 (3)	0.9947 ± 0.0028 (1)	0.7474 ± 0.0179 (4)	0.9929 ± 0.0044 (3)	0.9900 ± 0.0082 (3)
	Balanced ERM	0.9944 ± 0.0027 (1)	0.9947 ± 0.0028 (1)	0.9292 ± 0.0068 (2)	0.9939 ± 0.0033 (1)	0.9911 ± 0.0036 (2)
	Undersample	0.9706 ± 0.0119 (4)	0.9704 ± 0.0123 (4)	0.9211 ± 0.0142 (3)	0.9749 ± 0.0112 (4)	0.9855 ± 0.0147 (4)
	SMOTE	0.9944 ± 0.0027 (1)	0.9941 ± 0.0032 (3)	0.9301 ± 0.0066 (1)	0.9936 ± 0.0037 (2)	0.9937 ± 0.0039 (1)
chemo-a1a	ERM	0.7998 ± 0.1326 (5)	0.8263 ± 0.1076 (5)	0.7882 ± 0.1002 (5)	0.8337 ± 0.0915 (5)	0.8929 ± 0.0628 (3)
	Plugin	0.9121 ± 0.0597 (1)	0.9051 ± 0.0618 (1)	0.7927 ± 0.0308 (4)	0.9195 ± 0.0609 (1)	0.8973 ± 0.0618 (2)
	Balanced ERM	0.8952 ± 0.0687 (3)	0.9014 ± 0.0617 (2)	0.8888 ± 0.0671 (3)	0.8803 ± 0.0557 (4)	0.9409 ± 0.0316 (1)
	Undersample	0.9038 ± 0.0440 (2)	0.8989 ± 0.0486 (3)	0.9028 ± 0.0562 (1)	0.9083 ± 0.0391 (2)	0.8893 ± 0.0620 (4)
	SMOTE	0.8882 ± 0.0427 (4)	0.8703 ± 0.0492 (4)	0.8970 ± 0.0640 (2)	0.9060 ± 0.0558 (3)	0.8604 ± 0.0530 (5)
chemo-pde5	ERM	0.9097 ± 0.0773 (5)	0.9018 ± 0.0622 (5)	0.8980 ± 0.0867 (4)	0.9267 ± 0.0687 (4)	0.9305 ± 0.0681 (3)
	Plugin	0.9610 ± 0.0403 (1)	0.9321 ± 0.0650 (1)	0.7953 ± 0.0371 (5)	0.9625 ± 0.0351 (1)	0.9533 ± 0.0559 (1)
	Balanced ERM	0.9418 ± 0.0581 (2)	0.9278 ± 0.0705 (2)	0.9484 ± 0.0438 (2)	0.9390 ± 0.0623 (2)	0.9388 ± 0.0470 (2)
	Undersample	0.9270 ± 0.0381 (4)	0.9226 ± 0.0359 (3)	0.9260 ± 0.0356 (3)	0.9264 ± 0.0366 (5)	0.9266 ± 0.0425 (4)
	SMOTE	0.9379 ± 0.0638 (3)	0.9222 ± 0.0826 (4)	0.9501 ± 0.0410 (1)	0.9352 ± 0.0632 (3)	0.9264 ± 0.0810 (5)
covtype-binary	ERM	0.6629 ± 0.0168 (5)	0.6524 ± 0.0160 (5)	0.4229 ± 0.0137 (5)	0.6509 ± 0.0205 (5)	0.6733 ± 0.0154 (5)
	Plugin	0.8781 ± 0.0045 (4)	0.8836 ± 0.0047 (2)	0.8065 ± 0.0041 (4)	0.8737 ± 0.0051 (4)	0.8547 ± 0.0067 (4)
	Balanced ERM	0.8841 ± 0.0041 (3)	0.8839 ± 0.0049 (1)	0.8791 ± 0.0045 (2)	0.8839 ± 0.0040 (1)	0.8834 ± 0.0040 (3)
	Undersample	0.8848 ± 0.0045 (1)	0.8824 ± 0.0047 (4)	0.8789 ± 0.0046 (3)	0.8839 ± 0.0044 (1)	0.8843 ± 0.0043 (2)
	SMOTE	0.8846 ± 0.0037 (2)	0.8832 ± 0.0048 (3)	0.8798 ± 0.0048 (1)	0.8838 ± 0.0034 (3)	0.8855 ± 0.0041 (1)
german	ERM	0.6505 ± 0.0360 (5)	0.6442 ± 0.0322 (5)	0.6402 ± 0.0278 (5)	0.6502 ± 0.0301 (5)	0.6449 ± 0.0365 (5)
	Plugin	0.7109 ± 0.0267 (2)	0.7017 ± 0.0333 (4)	0.7074 ± 0.0275 (2)	0.7114 ± 0.0316 (2)	0.7178 ± 0.0258 (1)
	Balanced ERM	0.7071 ± 0.0337 (4)	0.7029 ± 0.0364 (3)	0.7047 ± 0.0342 (3)	0.7056 ± 0.0328 (4)	0.7047 ± 0.0372 (3)
	Undersample	0.7082 ± 0.0348 (3)	0.7140 ± 0.0291 (1)	0.7020 ± 0.0360 (4)	0.7059 ± 0.0359 (3)	0.7032 ± 0.0329 (4)
	SMOTE	0.7163 ± 0.0322 (1)	0.7063 ± 0.0321 (2)	0.7192 ± 0.0282 (1)	0.7185 ± 0.0288 (1)	0.7120 ± 0.0264 (2)
kddcup08	ERM	0.5147 ± 0.0312 (5)	0.5182 ± 0.0528 (5)	0.0087 ± 0.0276 (5)	0.4209 ± 0.0372 (5)	0.4945 ± 0.0299 (5)
	Plugin	0.8434 ± 0.0185 (1)	0.8261 ± 0.0245 (2)	0.7149 ± 0.0158 (4)	0.8349 ± 0.0197 (1)	0.8253 ± 0.0153 (1)
	Balanced ERM	0.8239 ± 0.0181 (3)	0.8232 ± 0.0160 (3)	0.8229 ± 0.0129 (2)	0.8268 ± 0.0191 (2)	0.8248 ± 0.0163 (2)
	Undersample	0.8052 ± 0.0187 (4)	0.8022 ± 0.0171 (4)	0.8114 ± 0.0164 (3)	0.8076 ± 0.0202 (4)	0.8026 ± 0.0199 (4)
	SMOTE	0.8261 ± 0.0184 (2)	0.8272 ± 0.0231 (1)	0.8257 ± 0.0147 (1)	0.8257 ± 0.0202 (3)	0.8208 ± 0.0204 (3)
kddcup98	ERM	0.0000 ± 0.0000 (5)	0.0000 ± 0.0000 (5)	0.0000 ± 0.0000 (5)	0.0000 ± 0.0000 (5)	0.0000 ± 0.0000 (5)
	Plugin	0.5856 ± 0.0059 (1)	0.5850 ± 0.0072 (1)	0.5838 ± 0.0052 (3)	0.5842 ± 0.0052 (2)	0.4174 ± 0.0722 (4)
	Balanced ERM	0.5846 ± 0.0065 (2)	0.5848 ± 0.0071 (2)	0.5846 ± 0.0068 (1)	0.5848 ± 0.0070 (1)	0.5798 ± 0.0070 (3)
	Undersample	0.5844 ± 0.0070 (3)	0.5840 ± 0.0067 (3)	0.5840 ± 0.0066 (2)	0.5841 ± 0.0066 (3)	0.5808 ± 0.0069 (1)
	SMOTE	0.5833 ± 0.0076 (4)	0.5840 ± 0.0073 (3)	0.5838 ± 0.0080 (3)	0.5832 ± 0.0076 (4)	0.5805 ± 0.0061 (2)
letter	ERM	0.8134 ± 0.0228 (5)	0.8004 ± 0.0241 (5)	0.0000 ± 0.0000 (5)	0.8107 ± 0.0199 (5)	0.8194 ± 0.0238 (5)
	Plugin	0.9383 ± 0.0148 (4)	0.9477 ± 0.0078 (3)	0.7197 ± 0.0067 (4)	0.9368 ± 0.0130 (4)	0.9227 ± 0.0174 (4)
	Balanced ERM	0.9526 ± 0.0058 (2)	0.9473 ± 0.0092 (4)	0.9376 ± 0.0058 (1)	0.9525 ± 0.0055 (2)	0.9539 ± 0.0059 (3)
	Undersample	0.9517 ± 0.0051 (3)	0.9482 ± 0.0070 (2)	0.9359 ± 0.0069 (3)	0.9522 ± 0.0050 (3)	0.9540 ± 0.0057 (2)
	SMOTE	0.9546 ± 0.0056 (1)	0.9493 ± 0.0075 (1)	0.9368 ± 0.0070 (2)	0.9535 ± 0.0060 (1)	0.9558 ± 0.0060 (1)
optdigits	ERM	0.9876 ± 0.0085 (5)	0.9871 ± 0.0081 (5)	0.9862 ± 0.0094 (4)	0.9863 ± 0.0076 (5)	0.9885 ± 0.0064 (4)
	Plugin	0.9893 ± 0.0056 (3)	0.9895 ± 0.0067 (1)	0.8628 ± 0.0048 (5)	0.9873 ± 0.0053 (4)	0.9908 ± 0.0065 (1)
	Balanced ERM	0.9899 ± 0.0067 (1)	0.9892 ± 0.0067 (2)	0.9925 ± 0.0035 (1)	0.9902 ± 0.0071 (1)	0.9895 ± 0.0073 (3)
	Undersample	0.9885 ± 0.0059 (4)	0.9880 ± 0.0078 (4)	0.9923 ± 0.0033 (3)	0.9887 ± 0.0065 (3)	0.9881 ± 0.0071 (5)
	SMOTE	0.9898 ± 0.0070 (2)	0.9883 ± 0.0072 (3)	0.9924 ± 0.0043 (2)	0.9899 ± 0.0065 (2)	0.9898 ± 0.0070 (2)
pendigits	ERM	0.9158 ± 0.0145 (5)	0.9008 ± 0.0166 (5)	0.5619 ± 0.0259 (5)	0.9129 ± 0.0165 (5)	0.9193 ± 0.0146 (5)
	Plugin	0.9555 ± 0.0075 (4)	0.9498 ± 0.0061 (3)	0.7790 ± 0.0090 (4)	0.9509 ± 0.0066 (4)	0.9507 ± 0.0095 (4)
	Balanced ERM	0.9596 ± 0.0049 (2)	0.9495 ± 0.0059 (4)	0.9316 ± 0.0044 (2)	0.9595 ± 0.0053 (1)	0.9601 ± 0.0052 (2)
	Undersample	0.9573 ± 0.0056 (3)	0.9517 ± 0.0072 (1)	0.9306 ± 0.0056 (3)	0.9582 ± 0.0055 (3)	0.9589 ± 0.0061 (3)
	SMOTE	0.9597 ± 0.0061 (1)	0.9514 ± 0.0043 (2)	0.9326 ± 0.0048 (1)	0.9590 ± 0.0064 (2)	0.9615 ± 0.0057 (1)
satimage	ERM	0.1520 ± 0.0311 (5)	0.1252 ± 0.0536 (5)	0.0000 ± 0.0000 (5)	0.0792 ± 0.0476 (5)	0.0000 ± 0.0000 (5)
	Plugin	0.6868 ± 0.0128 (4)	0.7180 ± 0.0132 (1)	0.6683 ± 0.0087 (4)	0.6923 ± 0.0060 (4)	0.5893 ± 0.0895 (4)
	Balanced ERM	0.7178 ± 0.0129 (1)	0.7179 ± 0.0131 (2)	0.7149 ± 0.0138 (1)	0.7167 ± 0.0135 (1)	0.6981 ± 0.0078 (3)
	Undersample	0.7131 ± 0.0213 (3)	0.7133 ± 0.0211 (4)	0.7086 ± 0.0190 (3)	0.7084 ± 0.0192 (3)	0.7158 ± 0.0124 (2)
	SMOTE	0.7168 ± 0.0134 (2)	0.7167 ± 0.0138 (3)	0.7100 ± 0.0138 (2)	0.7105 ± 0.0162 (2)	0.7230 ± 0.0117 (1)
segment	ERM	0.9962 ± 0.0049 (4)	0.9962 ± 0.0049 (2)	0.9970 ± 0.0039 (1)	0.9969 ± 0.0040 (3)	0.9984 ± 0.0031 (1)
	Plugin	0.9962 ± 0.0049 (4)	0.9960 ± 0.0048 (4)	0.9800 ± 0.0065 (5)	0.9975 ± 0.0032 (1)	0.9973 ± 0.0036 (3)
	Balanced ERM	0.9978 ± 0.0035 (1)	0.9960 ± 0.0052 (4)	0.9970 ± 0.0039 (1)	0.9969 ± 0.0040 (3)	0.9978 ± 0.0037 (2)
	Undersample	0.9969 ± 0.0043 (2)	0.9961 ± 0.0046 (3)	0.9970 ± 0.0038 (1)	0.9970 ± 0.0046 (2)	0.9957 ± 0.0041 (5)
	SMOTE	0.9969 ± 0.0039 (2)	0.9963 ± 0.0043 (1)	0.9970 ± 0.0038 (1)	0.9968 ± 0.0038 (5)	0.9971 ± 0.0043 (4)
shuttle	ERM	0.2014 ± 0.2147 (5)	0.0000 ± 0.0000 (5)	0.0000 ± 0.0000 (5)	0.0000 ± 0.0000 (5)	0.1342 ± 0.2160 (5)
	Plugin	0.5653 ± 0.2060 (3)	0.6163 ± 0.0787 (2)	0.5846 ± 0.0719 (4)	0.6052 ± 0.0613 (3)	0.5934 ± 0.0839 (3)
	Balanced ERM	0.5838 ± 0.1125 (2)	0.6163 ± 0.0788 (2)	0.6199 ± 0.0705 (2)	0.6046 ± 0.0953 (4)	0.5954 ± 0.0914 (2)
	Undersample	0.6407 ± 0.0931 (1)	0.6443 ± 0.0979 (1)	0.6353 ± 0.1075 (1)	0.6334 ± 0.1089 (1)	0.6470 ± 0.0999 (1)
	SMOTE	0.5594 ± 0.0970 (4)	0.6027 ± 0.0815 (4)	0.6146 ± 0.0673 (3)	0.6154 ± 0.0686 (2)	0.5646 ± 0.1839 (4)
spambase	ERM	0.9175 ± 0.0060 (5)	0.9073 ± 0.0097 (5)	0.8630 ± 0.0132 (5)	0.9164 ± 0.0071 (5)	0.9206 ± 0.0085 (5)
	Plugin	0.9247 ± 0.0066 (3)	0.9178 ± 0.0069 (2)	0.9033 ± 0.0086 (1)	0.9231 ± 0.0073 (3)	0.9268 ± 0.0082 (1)
	Balanced ERM	0.9251 ± 0.0065 (2)	0.9179 ± 0.0067 (1)	0.9005 ± 0.0073 (3)	0.9243 ± 0.0063 (2)	0.9231 ± 0.0080 (4)
	Undersample	0.9227 ± 0.0074 (4)	0.9152 ± 0.0074 (4)	0.8991 ± 0.0077 (4)	0.9213 ± 0.0074 (4)	0.9245 ± 0.0074 (3)
	SMOTE	0.9256 ± 0.0069 (1)	0.9174 ± 0.0068 (3)	0.9024 ± 0.0063 (2)	0.9252 ± 0.0063 (1)	0.9251 ± 0.0069 (2)
splice	ERM	0.9680 ± 0.0081 (2)	0.9665 ± 0.0063 (2)	0.9670 ± 0.0073 (1)	0.9700 ± 0.0074 (2)	0.9662 ± 0.0096 (5)
	Plugin	0.9629 ± 0.0088 (4)	0.9658 ± 0.0071 (3)	0.8847 ± 0.0112 (5)	0.9585 ± 0.0082 (5)	0.9702 ± 0.0072 (1)
	Balanced ERM	0.9664 ± 0.0077 (3)	0.9657 ± 0.0070 (4)	0.9597 ± 0.0070 (3)	0.9670 ± 0.0065 (3)	0.9685 ± 0.0078 (3)
	Undersample	0.9618 ± 0.0066 (5)	0.9610 ± 0.0067 (5)	0.9580 ± 0.0055 (4)	0.9638 ± 0.0057 (4)	0.9677 ± 0.0086 (4)
	SMOTE	0.9691 ± 0.0060 (1)	0.9674 ± 0.0069 (1)	0.9613 ± 0.0063 (2)	0.9706 ± 0.0064 (1)	0.9699 ± 0.0080 (2)
thyroid	ERM	0.8352 ± 0.0390 (5)	0.8360 ± 0.0383 (5)	0.4906 ± 0.0883 (5)	0.8382 ± 0.0358 (5)	0.8385 ± 0.0361 (5)
	Plugin	0.9667 ± 0.0144 (4)	0.9761 ± 0.0175 (1)	0.8127 ± 0.0076 (4)	0.9726 ± 0.0134 (4)	0.9837 ± 0.0045 (1)
	Balanced ERM	0.9765 ± 0.0190 (1)	0.9745 ± 0.0185 (2)	0.9746 ± 0.0078 (1)	0.9740 ± 0.0146 (2)	0.9757 ± 0.0190 (2)
	Undersample	0.9720 ± 0.0139 (2)	0.9716 ± 0.0164 (3)	0.9597 ± 0.0144 (3)	0.9742 ± 0.0157	

Statistical Consistency of Class Imbalance Methods

Table 7. Results for all algorithms on all data sets, in terms of AUC-ROC. For each loss function, the ranks of different algorithms are displayed in parentheses (i.e. ranks are column-relative within each data set block). Higher AUC values are better; lower ranks are better.

Dataset	Algorithm	Logistic	Exp	Square	Sq-Hinge	Hinge
abalone	ERM	0.7860 ± 0.0794 (4)	0.7943 ± 0.0712 (3)	0.8519 ± 0.0583 (1)	0.7839 ± 0.0912 (5)	0.6121 ± 0.0873 (4)
	Plugin	0.7696 ± 0.0640 (5)	0.7980 ± 0.0661 (1)	0.8498 ± 0.0560 (2)	0.7921 ± 0.0857 (4)	0.6075 ± 0.1185 (5)
	Balanced ERM	0.8082 ± 0.0654 (1)	0.7963 ± 0.0619 (2)	0.8228 ± 0.0570 (3)	0.8086 ± 0.0607 (2)	0.8217 ± 0.0628 (1)
	Undersample	0.7942 ± 0.0717 (3)	0.7806 ± 0.0789 (5)	0.8130 ± 0.0614 (5)	0.8117 ± 0.0695 (1)	0.8184 ± 0.0666 (2)
	SMOTE	0.8018 ± 0.0612 (2)	0.7826 ± 0.0638 (4)	0.8169 ± 0.0536 (4)	0.8033 ± 0.0608 (3)	0.8104 ± 0.0648 (3)
car	ERM	0.9985 ± 0.0011 (1)	0.9985 ± 0.0012 (1)	0.9961 ± 0.0020 (4)	0.9984 ± 0.0013 (3)	0.9979 ± 0.0009 (3)
	Plugin	0.9984 ± 0.0011 (4)	0.9985 ± 0.0012 (1)	0.9964 ± 0.0022 (3)	0.9984 ± 0.0011 (3)	0.9980 ± 0.0011 (2)
	Balanced ERM	0.9985 ± 0.0010 (1)	0.9985 ± 0.0012 (1)	0.9980 ± 0.0015 (1)	0.9985 ± 0.0011 (1)	0.9981 ± 0.0019 (1)
	Undersample	0.9936 ± 0.0051 (5)	0.9933 ± 0.0051 (5)	0.9937 ± 0.0051 (5)	0.9935 ± 0.0061 (5)	0.9957 ± 0.0038 (5)
	SMOTE	0.9985 ± 0.0010 (1)	0.9984 ± 0.0012 (4)	0.9965 ± 0.0026 (2)	0.9985 ± 0.0010 (1)	0.9979 ± 0.0016 (3)
chemo-a1a	ERM	0.9717 ± 0.0324 (4)	0.9762 ± 0.0272 (3)	0.9315 ± 0.0604 (5)	0.9818 ± 0.0213 (1)	0.9803 ± 0.0213 (2)
	Plugin	0.9842 ± 0.0175 (1)	0.9796 ± 0.0123 (1)	0.9701 ± 0.0288 (3)	0.9817 ± 0.0206 (2)	0.9807 ± 0.0174 (1)
	Balanced ERM	0.9768 ± 0.0132 (3)	0.9759 ± 0.0130 (4)	0.9769 ± 0.0120 (2)	0.9750 ± 0.0116 (4)	0.9791 ± 0.0112 (3)
	Undersample	0.9685 ± 0.0144 (5)	0.9633 ± 0.0265 (5)	0.9665 ± 0.0293 (4)	0.9688 ± 0.0189 (5)	0.9677 ± 0.0147 (5)
	SMOTE	0.9796 ± 0.0104 (2)	0.9765 ± 0.0069 (2)	0.9830 ± 0.0116 (1)	0.9807 ± 0.0120 (3)	0.9721 ± 0.0098 (4)
chemo-pde5	ERM	0.9845 ± 0.0267 (2)	0.9876 ± 0.0247 (1)	0.9615 ± 0.0486 (5)	0.9880 ± 0.0239 (1)	0.9852 ± 0.0233 (1)
	Plugin	0.9868 ± 0.0260 (1)	0.9801 ± 0.0321 (3)	0.9784 ± 0.0366 (4)	0.9817 ± 0.0438 (3)	0.9829 ± 0.0303 (2)
	Balanced ERM	0.9802 ± 0.0321 (3)	0.9796 ± 0.0326 (4)	0.9828 ± 0.0284 (1)	0.9821 ± 0.0315 (2)	0.9803 ± 0.0315 (4)
	Undersample	0.9785 ± 0.0253 (5)	0.9778 ± 0.0252 (5)	0.9797 ± 0.0248 (3)	0.9797 ± 0.0249 (5)	0.9793 ± 0.0248 (5)
	SMOTE	0.9798 ± 0.0316 (4)	0.9828 ± 0.0293 (2)	0.9818 ± 0.0286 (2)	0.9812 ± 0.0307 (4)	0.9815 ± 0.0253 (3)
covtype-binary	ERM	0.9472 ± 0.0031 (5)	0.9483 ± 0.0031 (1)	0.9298 ± 0.0037 (5)	0.9475 ± 0.0033 (4)	0.9396 ± 0.0038 (4)
	Plugin	0.9482 ± 0.0032 (4)	0.9483 ± 0.0029 (1)	0.9299 ± 0.0038 (4)	0.9475 ± 0.0033 (4)	0.9386 ± 0.0040 (5)
	Balanced ERM	0.9489 ± 0.0027 (1)	0.9483 ± 0.0029 (1)	0.9451 ± 0.0027 (2)	0.9486 ± 0.0026 (1)	0.9475 ± 0.0027 (1)
	Undersample	0.9485 ± 0.0027 (3)	0.9477 ± 0.0030 (5)	0.9449 ± 0.0026 (3)	0.9481 ± 0.0027 (3)	0.9466 ± 0.0029 (3)
	SMOTE	0.9487 ± 0.0026 (2)	0.9482 ± 0.0029 (4)	0.9452 ± 0.0025 (1)	0.9484 ± 0.0025 (2)	0.9469 ± 0.0027 (2)
german	ERM	0.7835 ± 0.0305 (2)	0.7841 ± 0.0313 (1)	0.7834 ± 0.0308 (4)	0.7831 ± 0.0314 (4)	0.7803 ± 0.0324 (3)
	Plugin	0.7851 ± 0.0286 (1)	0.7824 ± 0.0293 (2)	0.7852 ± 0.0300 (1)	0.7855 ± 0.0298 (1)	0.7840 ± 0.0319 (1)
	Balanced ERM	0.7835 ± 0.0321 (2)	0.7806 ± 0.0304 (3)	0.7845 ± 0.0313 (3)	0.7840 ± 0.0310 (3)	0.7828 ± 0.0317 (2)
	Undersample	0.7787 ± 0.0327 (5)	0.7799 ± 0.0254 (4)	0.7788 ± 0.0324 (5)	0.7809 ± 0.0321 (5)	0.7801 ± 0.0270 (5)
	SMOTE	0.7816 ± 0.0279 (4)	0.7797 ± 0.0309 (5)	0.7850 ± 0.0272 (2)	0.7841 ± 0.0282 (2)	0.7802 ± 0.0246 (4)
kddcup08	ERM	0.9218 ± 0.0138 (2)	0.9128 ± 0.0163 (4)	0.8681 ± 0.0150 (5)	0.9216 ± 0.0137 (2)	0.9083 ± 0.0139 (3)
	Plugin	0.9220 ± 0.0112 (1)	0.9161 ± 0.0183 (1)	0.8720 ± 0.0125 (4)	0.9230 ± 0.0128 (1)	0.9106 ± 0.0168 (1)
	Balanced ERM	0.9102 ± 0.0139 (4)	0.9136 ± 0.0148 (3)	0.9084 ± 0.0133 (2)	0.9132 ± 0.0140 (4)	0.9073 ± 0.0139 (4)
	Undersample	0.8898 ± 0.0153 (5)	0.8867 ± 0.0126 (5)	0.8930 ± 0.0120 (3)	0.8924 ± 0.0155 (5)	0.8844 ± 0.0154 (5)
	SMOTE	0.9153 ± 0.0160 (3)	0.9153 ± 0.0169 (2)	0.9089 ± 0.0122 (1)	0.9160 ± 0.0154 (3)	0.9098 ± 0.0121 (2)
kddcup98	ERM	0.6147 ± 0.0076 (1)	0.6148 ± 0.0077 (1)	0.6145 ± 0.0076 (3)	0.6145 ± 0.0076 (2)	0.5110 ± 0.0439 (5)
	Plugin	0.6147 ± 0.0078 (1)	0.6145 ± 0.0081 (3)	0.6146 ± 0.0077 (2)	0.6145 ± 0.0076 (2)	0.5328 ± 0.0222 (4)
	Balanced ERM	0.6143 ± 0.0077 (3)	0.6146 ± 0.0080 (2)	0.6147 ± 0.0076 (1)	0.6147 ± 0.0077 (1)	0.6092 ± 0.0071 (3)
	Undersample	0.6142 ± 0.0071 (4)	0.6136 ± 0.0070 (4)	0.6136 ± 0.0069 (4)	0.6136 ± 0.0069 (4)	0.6096 ± 0.0072 (1)
	SMOTE	0.6119 ± 0.0089 (5)	0.6124 ± 0.0083 (5)	0.6124 ± 0.0089 (5)	0.6120 ± 0.0088 (5)	0.6094 ± 0.0069 (2)
letter	ERM	0.9866 ± 0.0041 (5)	0.9882 ± 0.0023 (3)	0.9741 ± 0.0045 (4)	0.9864 ± 0.0040 (4)	0.9823 ± 0.0061 (4)
	Plugin	0.9867 ± 0.0042 (4)	0.9883 ± 0.0026 (1)	0.9741 ± 0.0045 (4)	0.9863 ± 0.0043 (5)	0.9823 ± 0.0061 (4)
	Balanced ERM	0.9884 ± 0.0023 (1)	0.9882 ± 0.0027 (3)	0.9847 ± 0.0020 (2)	0.9883 ± 0.0021 (1)	0.9874 ± 0.0022 (1)
	Undersample	0.9877 ± 0.0021 (3)	0.9877 ± 0.0022 (5)	0.9846 ± 0.0019 (3)	0.9876 ± 0.0020 (3)	0.9864 ± 0.0022 (3)
	SMOTE	0.9884 ± 0.0022 (1)	0.9883 ± 0.0025 (1)	0.9848 ± 0.0020 (1)	0.9883 ± 0.0022 (1)	0.9870 ± 0.0023 (2)
optdigits	ERM	0.9992 ± 0.0010 (5)	0.9992 ± 0.0008 (4)	0.9997 ± 0.0003 (1)	0.9992 ± 0.0008 (4)	0.9995 ± 0.0005 (3)
	Plugin	0.9995 ± 0.0005 (2)	0.9994 ± 0.0006 (2)	0.9997 ± 0.0003 (1)	0.9991 ± 0.0008 (5)	0.9997 ± 0.0004 (1)
	Balanced ERM	0.9995 ± 0.0006 (2)	0.9994 ± 0.0007 (2)	0.9996 ± 0.0004 (3)	0.9995 ± 0.0007 (2)	0.9995 ± 0.0006 (3)
	Undersample	0.9995 ± 0.0006 (2)	0.9991 ± 0.0010 (5)	0.9996 ± 0.0003 (3)	0.9995 ± 0.0007 (2)	0.9993 ± 0.0008 (5)
	SMOTE	0.9997 ± 0.0002 (1)	0.9995 ± 0.0006 (1)	0.9996 ± 0.0004 (3)	0.9997 ± 0.0002 (1)	0.9996 ± 0.0004 (2)
pendigits	ERM	0.9911 ± 0.0023 (4)	0.9900 ± 0.0015 (4)	0.9835 ± 0.0032 (5)	0.9910 ± 0.0025 (4)	0.9892 ± 0.0041 (5)
	Plugin	0.9913 ± 0.0021 (1)	0.9901 ± 0.0016 (2)	0.9842 ± 0.0035 (4)	0.9913 ± 0.0022 (1)	0.9901 ± 0.0036 (3)
	Balanced ERM	0.9912 ± 0.0016 (3)	0.9901 ± 0.0016 (2)	0.9869 ± 0.0026 (1)	0.9911 ± 0.0015 (2)	0.9907 ± 0.0016 (1)
	Undersample	0.9909 ± 0.0017 (5)	0.9895 ± 0.0018 (5)	0.9863 ± 0.0028 (3)	0.9907 ± 0.0016 (5)	0.9901 ± 0.0016 (3)
	SMOTE	0.9913 ± 0.0016 (1)	0.9906 ± 0.0015 (1)	0.9868 ± 0.0025 (2)	0.9911 ± 0.0016 (2)	0.9905 ± 0.0017 (2)
satimage	ERM	0.7754 ± 0.0134 (1)	0.7662 ± 0.0146 (1)	0.7554 ± 0.0117 (1)	0.7742 ± 0.0144 (1)	0.7197 ± 0.0182 (4)
	Plugin	0.7532 ± 0.0166 (2)	0.7275 ± 0.0098 (2)	0.7377 ± 0.0107 (2)	0.7723 ± 0.0066 (2)	0.7154 ± 0.0105 (5)
	Balanced ERM	0.7274 ± 0.0098 (3)	0.7274 ± 0.0099 (4)	0.7291 ± 0.0127 (5)	0.7275 ± 0.0098 (5)	0.7660 ± 0.0111 (1)
	Undersample	0.7270 ± 0.0099 (5)	0.7269 ± 0.0099 (5)	0.7309 ± 0.0162 (4)	0.7349 ± 0.0187 (3)	0.7350 ± 0.0195 (2)
	SMOTE	0.7274 ± 0.0094 (3)	0.7275 ± 0.0094 (2)	0.7312 ± 0.0154 (3)	0.7341 ± 0.0181 (4)	0.7273 ± 0.0095 (3)
segment	ERM	0.9999 ± 0.0003 (1)	0.9999 ± 0.0003 (1)	0.9977 ± 0.0047 (5)	0.9999 ± 0.0003 (1)	0.9998 ± 0.0006 (1)
	Plugin	0.9999 ± 0.0003 (1)	0.9999 ± 0.0003 (1)	0.9979 ± 0.0047 (4)	0.9999 ± 0.0003 (1)	0.9998 ± 0.0006 (1)
	Balanced ERM	0.9999 ± 0.0003 (1)	0.9999 ± 0.0003 (1)	0.9982 ± 0.0047 (1)	0.9998 ± 0.0005 (3)	0.9994 ± 0.0017 (4)
	Undersample	0.9997 ± 0.0008 (5)	0.9997 ± 0.0008 (5)	0.9982 ± 0.0046 (1)	0.9997 ± 0.0008 (5)	0.9996 ± 0.0011 (3)
	SMOTE	0.9999 ± 0.0004 (1)	0.9999 ± 0.0003 (1)	0.9981 ± 0.0045 (3)	0.9998 ± 0.0004 (3)	0.9993 ± 0.0021 (5)
shuttle	ERM	0.5813 ± 0.1313 (4)	0.7525 ± 0.0678 (1)	0.5667 ± 0.1468 (4)	0.5667 ± 0.1468 (4)	0.5839 ± 0.1536 (4)
	Plugin	0.5640 ± 0.1076 (5)	0.6832 ± 0.1305 (2)	0.5586 ± 0.1339 (5)	0.5586 ± 0.1339 (5)	0.5698 ± 0.1477 (5)
	Balanced ERM	0.6729 ± 0.1217 (2)	0.6826 ± 0.1268 (3)	0.6628 ± 0.1632 (3)	0.6961 ± 0.1351 (1)	0.7091 ± 0.1098 (1)
	Undersample	0.6745 ± 0.1388 (1)	0.6734 ± 0.1412 (4)	0.6816 ± 0.1360 (1)	0.6751 ± 0.1376 (2)	0.6754 ± 0.1490 (2)
	SMOTE	0.6292 ± 0.1145 (3)	0.6311 ± 0.1405 (5)	0.6424 ± 0.1488 (3)	0.6415 ± 0.1480 (3)	0.6689 ± 0.1475 (3)
spambase	ERM	0.9714 ± 0.0047 (2)	0.9697 ± 0.0046 (1)	0.9507 ± 0.0065 (4)	0.9712 ± 0.0048 (1)	0.9716 ± 0.0053 (2)
	Plugin	0.9713 ± 0.0046 (4)	0.9694 ± 0.0044 (3)	0.9505 ± 0.0063 (5)	0.9709 ± 0.0047 (3)	0.9711 ± 0.0055 (3)
	Balanced ERM	0.9718 ± 0.0048 (1)	0.9694 ± 0.0044 (3)	0.9517 ± 0.0061 (3)	0.9712 ± 0.0043 (1)	0.9707 ± 0.0051 (4)
	Undersample	0.9709 ± 0.0049 (5)	0.9688 ± 0.0048 (5)	0.9518 ± 0.0064 (1)	0.9706 ± 0.0046 (5)	0.9706 ± 0.0055 (5)
	SMOTE	0.9714 ± 0.0046 (2)	0.9695 ± 0.0045 (2)	0.9518 ± 0.0062 (1)	0.9709 ± 0.0041 (3)	0.9718 ± 0.0049 (1)
splice	ERM	0.9939 ± 0.0027 (1)	0.9931 ± 0.0030 (4)	0.9935 ± 0.0031 (4)	0.9938 ± 0.0028 (3)	0.9934 ± 0.0032 (4)
	Plugin	0.9932 ± 0.0030 (4)	0.9935 ± 0.0028 (1)	0.9933 ± 0.0024 (5)	0.9924 ± 0.0030 (5)	0.9937 ± 0.0029 (3)
	Balanced ERM	0.9938 ± 0.0025 (2)	0.9935 ± 0.0028 (1)	0.9944 ± 0.0025 (1)	0.9939 ± 0.0029 (2)	0.9934 ± 0.0037 (4)
	Undersample	0.9932 ± 0.0033 (4)	0.9929 ± 0.0030 (5)	0.9939 ± 0.0026 (3)	0.9938 ± 0.0028 (3)	0.9939 ± 0.0029 (2)
	SMOTE	0.9938 ± 0.0026 (2)	0.9934 ± 0.0026 (3)	0.9941 ± 0.0025 (2)	0.9941 ± 0.0027 (1)	0.9943 ± 0.0027 (1)
thyroid	ERM	0.9925 ± 0.0107 (4)	0.9893 ± 0.0131 (5)	0.9957 ± 0.0010 (1)	0.9947 ± 0.0052 (4)	0.9960 ± 0.0022 (2)
	Plugin	0.9922 ± 0.0105 (5)	0.9953 ± 0.0034 (2)	0.9957 ± 0.0010 (1)	0.9945 ± 0.0052 (5)	0.9973 ± 0.0008 (1)
	Balanced ERM	0.9961 ± 0.0022 (1)	0.9951 ± 0.0036 (3)	0.9941 ± 0.0026 (4)	0.9958 ± 0.0021 (2)	0.9959 ± 0.0036 (3)
	Undersample					