

# Skip Context Tree Switching

## Supplementary Material

### 1 Proof of Theorem 1

The probability SkipCTS assigns to  $x_{1:n} \in \mathcal{X}^n$  is defined as  $\xi_{\text{SkipCTS}}(x_{1:n}) := \xi_\epsilon(x_{1:n})$ . We thus expand the loss  $-\log \xi_{\text{SkipCTS}}(x_{1:n})$  using Lemma 3:

$$\begin{aligned}
 -\log \xi_{\text{SkipCTS}}(x_{1:n}) &\leq -\log \left[ n^{-\ell(\mathcal{S})} \prod_{(c,l) \in \bar{\mathcal{S}}} \beta_{c,l}(\epsilon) \prod_{c \in \mathcal{S}} \alpha_c(x_{<n}) \rho_c(x_n | x_{<n}^c) \right] \\
 &= \ell(\mathcal{S}) \log n - \log \prod_{(c,l) \in \bar{\mathcal{S}}} \beta_{c,l}(\epsilon) - \log \left[ \prod_{c \in \mathcal{S}} \alpha_c(x_{<n}) \rho_c(x_n | x_{<n}^c) \right]. \tag{1}
 \end{aligned}$$

We now expand the rightmost term of this equation using Lemma 1:

$$\begin{aligned}
 -\log \left[ \prod_{c \in \mathcal{S}} \alpha_c(x_{<n}) \rho_c(x_n | x_{<n}^c) \right] &\leq -\log \left[ \frac{1}{n} \prod_{c \in \mathcal{S}} \alpha_c(\epsilon) \rho_c(x_{<n}^c) \rho_c(x_n | x_{<n}^c) \right] \\
 &= \log n - \log \prod_{c \in \mathcal{S}} \alpha_c(\epsilon) - \log \prod_{c \in \mathcal{S}} \rho_c(x_{1:n}^c) \\
 &= \log n - \log \prod_{c \in \mathcal{S}} \alpha_c(\epsilon) - \log \psi_{\mathcal{S}}(x_{1:n}),
 \end{aligned}$$

where the last equality follows from the definition of  $\psi_{\mathcal{S}}(x_{1:n})$ . Incorporating this result into Equation 1, we obtain

$$-\log \xi_{\text{SkipCTS}}(x_{1:n}) \leq [\ell(\mathcal{S}) + 1] \log n - \log \prod_{(c,l) \in \bar{\mathcal{S}}} \beta_{c,l}(\epsilon) - \log \prod_{c \in \mathcal{S}} \alpha_c(\epsilon) - \log \psi_{\mathcal{S}}(x_{1:n})$$

and hence

$$\begin{aligned}
 \mathcal{R}_n(\xi_{\text{SkipCTS}}, \{\psi_{\mathcal{S}}\}) &\leq [\ell(\mathcal{S}) + 1] \log n - \log \prod_{(c,l) \in \bar{\mathcal{S}}} \beta_{c,l}(\epsilon) - \log \prod_{c \in \mathcal{S}} \alpha_c(\epsilon) \\
 &= [\ell(\mathcal{S}) + 1] \log n + \Gamma_D^K(\mathcal{S})
 \end{aligned}$$

as desired.

|                     | K = 0   |        | K = 1   |       |      | K = 2   |       |      |
|---------------------|---------|--------|---------|-------|------|---------|-------|------|
|                     | Loss    | Speed  | Loss    | Speed | Sig. | Loss    | Speed | Sig. |
| ASTERIX             | 55.79   | 346.61 | 48.39   | 34.41 | ✓    | 38.30   | 7.35  | ✓    |
| BEAM RIDER          | 94.98   | 296.17 | 86.11   | 34.40 | ✓    | 101.63  | 7.01  | ✓    |
| PONG                | 7.78    | 346.79 | 6.83    | 37.96 | ✓    | 7.90    | 19.98 | ✓    |
| Q*BERT              | 7.04    | 330.02 | 6.46    | 39.92 | ✓    | 7.18    | 9.71  | ✓    |
| SEAQUEST            | 99.31   | 316.30 | 89.73   | 33.49 | ✓    | 78.07   | 6.85  | ✓    |
| ALIEN               | 59.13   | 276.87 | 53.92   | 35.23 | ✓    | 62.35   | 6.09  | ✓    |
| AMIDAR              | 11.84   | 317.90 | 10.75   | 38.60 | ✓    | 10.84   | 9.00  |      |
| ASSAULT             | 39.26   | 352.55 | 34.89   | 35.75 | ✓    | 34.84   | 10.42 |      |
| ASTEROIDS           | 24.25   | 352.44 | 23.99   | 35.47 | ✓    | 16.19   | 21.42 | ✓    |
| ATLANTIS            | 21.96   | 273.05 | 19.80   | 30.67 | ✓    | 19.98   | 12.69 |      |
| BANK HEIST          | 78.71   | 282.97 | 77.99   | 30.29 | ✓    | 72.97   | 6.58  | ✓    |
| BATTLE ZONE         | 193.35  | 240.58 | 179.92  | 28.37 | ✓    | 169.89  | 6.12  |      |
| BERZERK             | 54.76   | 278.57 | 50.80   | 29.41 | ✓    | 52.26   | 7.12  | ✓    |
| BOWLING             | 1.46    | 314.82 | 1.43    | 35.90 | ✓    | 1.58    | 17.73 |      |
| BOXING              | 194.76  | 275.75 | 192.38  | 39.84 | ✓    | 183.68  | 6.66  | ✓    |
| BREAKOUT            | 5.46    | 321.19 | 4.21    | 35.41 | ✓    | 5.56    | 18.81 | ✓    |
| CARNIVAL            | 31.38   | 301.71 | 24.67   | 32.76 | ✓    | 24.17   | 9.78  |      |
| CENTIPEDE           | 72.05   | 291.38 | 66.94   | 29.57 | ✓    | 66.97   | 5.61  |      |
| CHOPPER COMMAND     | 181.75  | 281.84 | 179.02  | 26.64 | ✓    | 155.57  | 5.65  | ✓    |
| CRAZY CLIMBER       | 42.61   | 357.73 | 35.18   | 34.54 | ✓    | 39.00   | 7.34  | ✓    |
| DEMON ATTACK        | 154.61  | 346.35 | 144.81  | 28.64 | ✓    | 143.56  | 8.26  |      |
| DOUBLE DUNK         | 192.16  | 297.47 | 190.52  | 27.85 | ✓    | 205.78  | 5.71  | ✓    |
| ELEVATOR ACTION     | 67.04   | 322.66 | 68.31   | 29.85 |      | 66.38   | 5.59  |      |
| ENDURO              | 276.54  | 274.30 | 247.83  | 26.53 | ✓    | 313.12  | 4.37  | ✓    |
| FISHING DERBY       | 111.75  | 254.84 | 101.51  | 29.34 | ✓    | 129.73  | 5.16  | ✓    |
| FREEWAY             | 6.71    | 260.11 | 4.32    | 37.14 | ✓    | 5.22    | 9.82  | ✓    |
| FROSTBITE           | 55.56   | 295.62 | 52.25   | 33.79 | ✓    | 60.35   | 6.09  | ✓    |
| GOPHER              | 23.18   | 358.41 | 19.14   | 36.11 | ✓    | 15.49   | 10.51 | ✓    |
| GRAVITAR            | 61.14   | 343.60 | 57.16   | 33.48 | ✓    | 53.98   | 8.18  | ✓    |
| H.E.R.O.            | 20.97   | 280.24 | 18.05   | 37.72 | ✓    | 19.75   | 7.96  | ✓    |
| ICE HOCKEY          | 98.01   | 296.70 | 97.22   | 29.65 | ✓    | 80.33   | 5.87  | ✓    |
| JAMES BOND          | 160.23  | 306.32 | 147.03  | 28.67 | ✓    | 162.47  | 5.28  | ✓    |
| JOURNEY ESCAPE      | 1104.44 | 182.08 | 1085.74 | 17.03 | ✓    | 1106.69 | 1.88  | ✓    |
| KANGAROO            | 17.11   | 319.69 | 16.52   | 37.63 | ✓    | 14.58   | 9.38  | ✓    |
| KRULL               | 143.16  | 261.17 | 129.05  | 29.41 | ✓    | 154.31  | 4.18  | ✓    |
| KUNG-FU MASTER      | 27.78   | 300.10 | 23.55   | 35.28 | ✓    | 25.38   | 6.70  | ✓    |
| MONTEZUMA'S REVENGE | 12.42   | 316.31 | 11.54   | 36.08 | ✓    | 12.86   | 7.35  | ✓    |
| MS. PACMAN          | 33.92   | 321.21 | 31.58   | 34.23 | ✓    | 34.09   | 5.30  | ✓    |
| NAME THIS GAME      | 54.61   | 301.56 | 45.73   | 32.97 | ✓    | 50.06   | 5.08  | ✓    |
| POOYAN              | 21.49   | 303.85 | 19.67   | 35.70 | ✓    | 21.26   | 8.66  | ✓    |
| PRIVATE EYE         | 95.03   | 290.91 | 83.69   | 31.46 | ✓    | 85.28   | 4.88  |      |
| RIVER RAID          | 83.65   | 283.98 | 74.08   | 32.50 | ✓    | 70.63   | 4.87  | ✓    |
| ROAD RUNNER         | 101.87  | 295.63 | 98.46   | 32.56 | ✓    | 104.66  | 6.05  | ✓    |
| ROBOTANK            | 206.08  | 262.14 | 179.70  | 28.84 | ✓    | 149.26  | 4.18  | ✓    |
| SKIING              | 73.33   | 273.06 | 71.77   | 32.64 | ✓    | 57.63   | 6.21  | ✓    |
| SPACE INVADERS      | 47.14   | 328.05 | 44.07   | 40.59 | ✓    | 45.02   | 5.81  |      |
| STAR GUNNER         | 133.17  | 363.04 | 111.79  | 36.55 | ✓    | 78.34   | 7.02  | ✓    |
| TENNIS              | 58.36   | 311.96 | 54.09   | 38.42 | ✓    | 49.38   | 5.93  | ✓    |
| TIME PILOT          | 173.83  | 334.52 | 163.52  | 30.20 | ✓    | 131.81  | 4.76  | ✓    |
| TUTANKHAM           | 79.16   | 330.27 | 68.17   | 44.46 | ✓    | 63.74   | 6.97  | ✓    |
| UP AND DOWN         | 204.05  | 231.64 | 196.29  | 37.32 | ✓    | 190.42  | 4.09  |      |
| VENTURE             | 24.13   | 349.33 | 22.38   | 51.87 | ✓    | 19.17   | 6.72  |      |
| VIDEO PINBALL       | 33.55   | 282.75 | 29.27   | 47.92 | ✓    | 40.83   | 6.48  | ✓    |
| WIZARD OF WOR       | 24.69   | 357.22 | 23.42   | 51.06 | ✓    | 23.66   | 7.52  |      |
| YAR'S REVENGE       | 112.39  | 324.94 | 104.83  | 36.36 | ✓    | 82.30   | 8.14  | ✓    |

Table 1: Prediction results for 55 Atari games. *Loss* corresponds to the per frame negative  $\log_2$  probability, averaged over the last 4500 frames of data of 10 trials. *Speed* corresponds to the average number of frames processed per second. *Sig.* indicates a statistically significant difference between  $K \in \{0, 1\}$  and  $K \in \{1, 2\}$ , respectively.