Effective Bayesian Modeling of Groups of Related Count Time Series (Supplementary Material)

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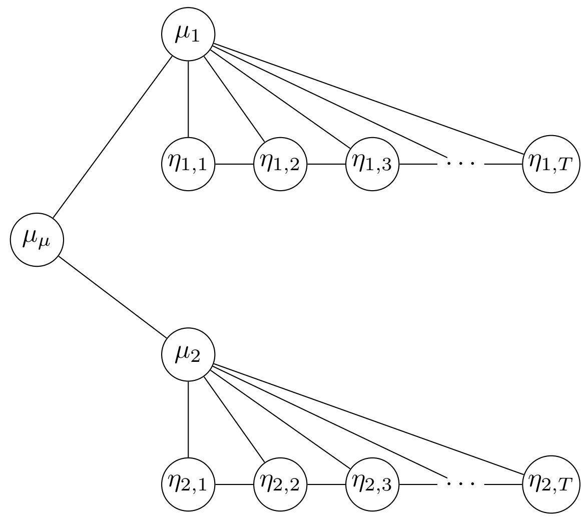
A. Process Hyper-Priors

The following hyperpriors are used for the hierarchical model described in section 2.3:

$$
\begin{aligned}
\bar{\alpha} &\sim U(0.001, 0.1), & \mu_\mu &\sim \mathcal{N}(0, 2^2),\\
\tau_\mu &\sim U(1, 10), & \kappa_\tau &\sim U(5, 10),\\
\beta_\tau &\sim U(2, 25), & \kappa_{0,\tau} &\sim U(1, 5),\\
\beta_{0,\tau} &\sim U(1, 10), & \kappa_\theta &\sim U(5, 10),\\
\beta_\theta &\sim U(2, 25), & \phi_+ &\sim U(1, 600),\\
\phi_- &\sim U(1, 50), & \bar{\boldsymbol{\theta}} &\sim \mathcal{N}(0, 1).
\end{aligned}
$$

B. Precision Matrix for Hierarchical GMRF Prior

The hierarchical model described in section 2.3 gives rise to a conditional Gaussian Markov random field (GMRF) prior over the global level of mean-reversion μ_μ, the series-specific levels of mean-reversion $\mu_\ell, \ell = 1, \ldots, L$, and the latent process log-means $\{\eta_{\ell,t}\}$. The GMRF prior structure for two time series is illustrated in the following graph:



In a GMRF, an edge in the graphical model corresponds to a non-zero entry in the precision matrix of the joint distribution over all variables. Hence, the precision matrix is very sparse: it has block diagonal structure, where each block corresponds to a single series. In the two-series example, assuming that each series has 4 observations, we have the following precision matrix:

$$
Q = \left[\begin{array}{ccccc|ccccc|c}
\tau_1 & -\tau_1\phi_1 & 0 & 0 & \tau_1\tilde{\phi}_1 & 0 & 0 & 0 & 0 & 0 & 0\\
-\tau_1\phi_1 & \tau_1\left(\phi_1^2+1\right) & -\tau_1\phi_1 & 0 & -\tau_1\tilde{\phi}_1^2 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -\tau_1\phi_1 & \tau_1\left(\phi_1^2+1\right) & -\tau_1\phi_1 & -\tau_1\tilde{\phi}_1^2 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -\tau_1\phi_1 & \tau_1 & \tau_1\tilde{\phi}_1 & 0 & 0 & 0 & 0 & 0 & 0\\
\tau_1\tilde{\phi}_1 & -\tau_1\tilde{\phi}_1^2 & -\tau_1\tilde{\phi}_1^2 & \tau_1\tilde{\phi}_1 & \tau_{\mu_1}+\tau_1\psi_{1,T} & 0 & 0 & 0 & 0 & 0 & -\tau_{\mu_1}\\
\hline
0 & 0 & 0 & 0 & 0 & \tau_2 & -\tau_2\phi_2 & 0 & 0 & \tau_2\tilde{\phi}_2 & 0\\
0 & 0 & 0 & 0 & 0 & -\tau_2\phi_2 & \tau_2\left(\phi_2^2+1\right) & -\tau_2\phi_2 & 0 & -\tau_2\tilde{\phi}_2^2 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -\tau_2\phi_2 & \tau_2\left(\phi_2^2+1\right) & -\tau_2\phi_2 & -\tau_2\tilde{\phi}_2^2 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\tau_2\phi_2 & \tau_2 & \tau_2\tilde{\phi}_2 & 0\\
0 & 0 & 0 & 0 & 0 & \tau_2\tilde{\phi}_2 & -\tau_2\tilde{\phi}_2^2 & -\tau_2\tilde{\phi}_2^2 & \tau_2\tilde{\phi}_2 & \tau_{\mu_2}+\tau_2\psi_{2,T} & -\tau_{\mu_2}\\
\hline
0 & 0 & 0 & 0 & -\tau_{\mu_1} & 0 & 0 & 0 & 0 & -\tau_{\mu_2} & \tau_{\mu_1}+\tau_{\mu_2}+\tau_{\mu_\mu}
\end{array}\right]
$$

where $T = 4$ (the number of periods), $\tilde{\phi}_1 = \phi_1 - 1$, $\tilde{\phi}_2 = \phi_2 - 1$, $\psi_{1,T} \equiv T - 2(T-1)\phi_1 + (T-2)\phi_1^2$,

$\psi_{2,T} \equiv T - 2(T-1)\phi_2 + (T-2)\phi_2^2$. The block structure is emphasized with dashed lines. The determinant of this matrix is $\tau_{\mu_\mu}\left(\tau_{\mu_1}\tau_1^T(\phi_1^2 - 1)\right)\left(\tau_{\mu_2}\tau_2^T(\phi_2^2 - 1)\right)$, which is useful for computing the probability of a variable configuration.