Here we prove Theorem 3 and 4 of Section 3.3 in the paper.

**Proof of Theorem 3**

*Proof.* We compute $P(n_{ij})$ only for the lower limit of the hypergeometric random variable support, i.e. $\max\{0, a_i + b_j - N\}$. In both cases $P(n_{ij})$ can be computed in $O(\max\{a_i, b_j\})$. All other probabilities are computed iteratively and thus their time expense is constant.

$$
\sum_{i=1}^{r} \sum_{j=1}^{c} \left( O(\max\{a_i, b_j\}) + \sum_{n_{ij}=0}^{\min\{a_i, b_j\}} O(1) \right) = \sum_{i=1}^{r} \sum_{j=1}^{c} O(\max\{a_i, b_j\}) = \sum_{i=1}^{r} O(\max\{ca_i, N\})
$$

$$
= O(\max\{cN, rN\})
$$

The above term is thus the computational complexity of the inner loop. Using the same machinery one can prove that:

$$
\sum_{j=1}^{c} \max\{a_i, b_j\} \sum_{n_{ij}=0}^{\min\{a_i, b_j\}} O(1) = \sum_{j=1}^{c} O(\max\{N, rb_j\})
$$

$$
= \sum_{j=1}^{c} O(\max\{a_i N, rb_j, b_j, rb_j^2\})
$$

$$
= O(\max\{ca_i N, a_i r N, r N^2\})
$$

The above term is thus the computational complexity of the inner loop. Using the same machinery one can prove that:

$$
\sum_{j=1}^{c} \sum_{i=1}^{r} \max\{a_i, b_j\} O(\max\{ca_i N, a_i r N, r N^2\}) = O(\max\{c^2 N^3, rc N^3\})
$$

\[\square\]