Supplementary Materials

A Proof of Theorem 2.1

Let b^* be an optimal solution for a given instance of the problem. We first examine properties of GreedyProcedure that hold for *arbitrary* b_0. We then show that there exists *some* b_0 in the enumeration step such that GreedyProcedure returns b with $f(b) \geq (1 - 1/e)f(b^*)$, which proves Theorem 2.1.

Let us fix an initial solution b_0, and analyze behavior of GreedyProcedure with input b_0. We denote by b_i the tentative solution b at the beginning of the ith iteration and denote by s_i and k_i s and k chosen in the ith iteration, respectively. Assume that GreedyProcedure first has not updated the tentative solution b in the Lth trial. Equivalently, let L be the minimum number such that $b_L = b_{L+1}$ and $b_i < b_{i+1}$ for $i = 1, \ldots, L-1$. Note that if such a situation never happens during the execution of GreedyProcedure, define L to be the number of iterations.

Lemma A.1. *Without loss of generality, we may assume that* $b(s_L) + k_L \leq b^*(s_L)$.

Proof. Suppose that $b(s_L)+k_L > b^*(s_L)$. Let us consider a modified instance in which the capacity of s_L is decreased to $b(s_L) + k_L - 1$. The optimal value is unchanged by this modification because b^* is still feasible and optimal. Furthermore, GreedyProcedure returns the same solution (with respect to same b_0). Thus it suffices to analyze the algorithm in the modified instance. Repeating this argument completes the proof of this lemma. □

Consider the ith iteration of the algorithm. For simplicity, we denote $\Delta(b_i, s_i, k_i)$ by Δ_i, $\delta(b_i, s_i, k_i)$ by δ_i, and $w(s_i)$ by w_i. Note that $f(b_i) = f(b_0) + \sum_{j=1}^{i-1} \Delta_i$ for $i = 1, \ldots, L$. Let $B' := B - w \cdot b_0$.

Lemma A.2. *For* $i = 1, \ldots, L$*, we have* $\Delta_i \geq \dfrac{w_i k_i}{B'} \left(f(b^*) - f(b_i)\right)$.

Proof. Let us denote $b_i \vee b^* = b_i + \sum \alpha_s \chi_s$, where the sum is taken over s in $\mathrm{supp}^+(b^* - b_i)$ and $\alpha_s := b^*(s) - b_i(s)$. Since $b_i \vee b^*(s)\chi_s = b_i + \alpha_s \chi_s$ for each $s \in \mathrm{supp}^+(b^* - b_i)$, (4) implies

$$\begin{aligned} f(b_i \vee b^*) &\leq f(b_i) + \sum_{s \in \mathrm{supp}^+(b^* - b_i)} \left(f(b_i + \alpha_s \chi_s) - f(b_i)\right) \\ &\leq f(b_i) + \sum_{s \in \mathrm{supp}^+(b^* - b_i)} w(s)\alpha_s \delta_i, \end{aligned}$$

where the last inequality follows from the fact that $(f(b_i + k\chi_s) - f(b_i))/(w(s)k) \leq \delta_i$ for all $s \in S$ and k. Since $\sum w(s)\alpha_s$ is at most B' and $f(b^*) \leq f(b_i \vee b^*)$ by the monotonicity of f, it holds that $f(b^*) \leq f(b_i) + \delta_i B'$. Therefore, we obtain $\delta_i \geq (f(b^*) - f(b_i))/B'$. □

Applying Lemma A.2 repeatedly, we have the following lemmas.

Lemma A.3. *For $i = 1, \ldots, L$, we have $f(b^*) - f(b_i) \leq (f(b^*) - f(b_0)) \cdot \prod_{j=1}^{i} (1 - w_j k_j / B')$.*

Proof. We prove this lemma by induction on i. For $i = 1$, then the inequality holds because $\Delta_1 \geq w_1 k_1 (f(b^*) - f(b_0))/B'$ by Lemma A.2. For $i > 1$, we have

$$
\begin{aligned}
& f(b^*) - f(b_0) - \sum_{j=1}^{i} \Delta_j = f(b^*) - f(b_0) - \sum_{j=1}^{i-1} \Delta_j - \Delta_i \\
& \leq f(b^*) - f(b_0) - \sum_{j=1}^{i-1} \Delta_j - \frac{w_i k_i}{B'} \left(f(b^*) - f(b_0) - \sum_{j=1}^{i-1} \Delta_j \right) \quad \text{(by Lemma A.2)} \\
& = \left(1 - \frac{w_i k_i}{B'}\right) \left(f(b^*) - f(b_0) - \sum_{j=1}^{i-1} \Delta_j \right) \\
& \leq \left(1 - \frac{w_i k_i}{B'}\right) \cdot (f(b^*) - f(b_0)) \cdot \prod_{j=1}^{i-1} \left(1 - \frac{w_j k_j}{B'}\right), \\
& \qquad \text{(by the induction hypothesis)}
\end{aligned}
$$

which completes the proof. □

Lemma A.4. *It holds that $f(b^*) - f(b_i) \leq (f(b^*) - f(b_0))/e$.*

Proof. Let $\varphi(x) = \ln(1 - x)$. Note that φ is concave in $[0, 1)$. By Jensen's inequality, we have $\sum_{i=1}^{L} \varphi(x_i)/L \leq \varphi(\sum_{i=1}^{L} x_i / L)$ for $x_1, \ldots, x_L \in [0, 1)$. Putting $x_i := w_i k_i / B'$, we obtain

$$
\frac{1}{L} \sum_{i=1}^{L} \ln \left(1 - \frac{w_i k_i}{B'}\right) \leq \ln \left(1 - \frac{1}{L} \sum_{i=1}^{L} \frac{w_i k_i}{B'}\right) \leq \ln \left(1 - \frac{1}{L}\right), \tag{1}
$$

where the last inequality follows since $\sum_{i=1}^{L} w_i k_i \geq B'$ and φ is a monotonically decreasing function. Thus we have

$$
\prod_{i=1}^{L} \left(1 - \frac{w_i k_i}{B'}\right) \leq \left(1 - \frac{1}{L}\right)^{L} \leq \frac{1}{e}. \tag{2}
$$

Combining this fact and Lemma A.3 completes the proof. □

We now move to proving that the greedy procedure returns a $(1-1/e)$-approximate solution for some b_0 in the enumeration step. If the optimal solution b^* satisfies $|\mathrm{supp}^+(b^*)| \leq 3$, it can be found in the enumeration step. Therefore, we assume that all the optimal solutions have more than three positive components. Let s_1^*, s_2^*, s_3^* be elements in S satisfying:

$$
s_i^* \in \operatorname*{argmax}_{s \in S \setminus \{s_1^*, \ldots, s_{i-1}^*\}} \Delta \left(\vee_{j=1}^{i-1} b^*(s_j^*) \chi_{s_j^*}, s, b^*(s) \right)
$$

for $i = 1, 2, 3$. Since the size of the support of b^* is more than three, such s_1^*, s_2^* and s_3^* clearly exist.

Lemma A.5. *For the feasible solution b_0 with the support $\{s_1^*, s_2^*, s_3^*\}$ satisfying $b_0(s_i^*) = b^*(s_i^*)$ for $1 \leq i \leq 3$, we have $\Delta_L \leq f(b_0)/3$.*

Proof. It follows that $\Delta_L = f(b_L + k_L \chi_{s_L}) - f(b_L) \leq f(b_L \vee b^*(s_L)\chi_{s_L}) - f(b_L)$ since $b_L(s_L) + k_L \leq b^*(s_L)$. By Lemma 2.2, Δ_L is at most $f(b^*(s_L)\chi_{s_L}) - f(0) = f(b^*(s_L)\chi_{s_L})$, and hence $\Delta_L \leq f(b^*(s_1^*)\chi_{s_1^*}) = \Delta(0, s_1^*, b^*(s_1^*))$ by the choice of s_1^*. Similarly, by the choices of s_2^* and s_3^*, we have $\Delta_L \leq \Delta(b^*(s_1^*)\chi_{s_1^*}, s_2^*, b^*(s_2^*))$ and $\Delta_L \leq \Delta(b^*(s_1^*)\chi_{s_1^*} \vee b^*(s_2^*)\chi_{s_2^*}, s_3^*, b^*(s_3^*))$. Adding these inequalities, we obtain $\Delta_L \leq f(b_0)/3$. □

We are now ready to prove Theorem 2.1. Since $f(b) \geq f(b_0) + \sum_{i=1}^{L} \Delta_i - \Delta_L$, Lemmas A.4 and A.5 imply that

$$\begin{aligned} f(b) &\geq (1 - 1/3) f(b_0) + (1 - 1/e)(f(b^*) - f(b_0)) \\ &\geq (1 - 1/e) f(b^*) \end{aligned}$$

for the initial solution b_0 described in Lemma A.5. This completes the proof of Theorem 2.1.

B Proof of Lemmas

B.1 Proof of Lemma 2.2

Proof. If $k \leq x(s)$ then both sides are 0. If $x(s) < k \leq y(s)$ then the inequality (3) is equivalent to $f(x \vee k\chi_s) - f(x) \geq 0$, which is valid by monotonicity. Lastly, if $k > y(s)$ then we have $f(x \vee k\chi_s) + f(y) \geq f(y \vee k\chi_s) + f(x)$ by submodularity, which directly implies (3). □

B.2 Proof of Lemma 2.3

Proof. We prove this lemma by induction on the size of $\mathrm{supp}^+(y - x)$. If $|\mathrm{supp}^+(y - x)| = 0$, that is, $x \vee y = x$, then (4) is trivial. Suppose that there is an index $s \in \mathrm{supp}^+(y - x)$. We define $y_1 = y - y(s)\chi_s$. Then $y = y_1 \vee y(s)\chi_s$ and $y_1 \wedge y(s)\chi_s = 0$ hold. By submodularity of $x \vee y_1$ and $x \vee y(s)\chi_s$, we have

$$f(x \vee y_1) + f(x \vee y(s)\chi_s) \geq f(x \vee y) + f((x \vee y_1) \wedge (x \vee y(s)\chi_s)).$$

Since it holds that

$$(x \vee y_1) \wedge (x \vee y(s)\chi_s) = x \vee (y_1 \wedge y(s)\chi_s) = x \vee 0 = x,$$

the above inequality implies that

$$f(x \vee y_1) + f(x \vee y(s)\chi_s) - f(x) \geq f(x \vee y).$$

Therefore, applying the induction hypothesis to x and y_1, we obtain (4). Thus the statement holds. □

B.3 Proof of Lemma 4.1

Proof. By simple algebra, we can easily check that

$$f(b+\chi_s) - f(b) = p_s^{(b(s)+1)} \sum_{t\in\Gamma(s)} \prod_{v\in\Gamma(t)} \prod_{i=1}^{b(v)} (1-p_v^{(i)}), \tag{3}$$

$$f(b+2\chi_s) - f(b+\chi_s) = (1-p_s^{(b(s)+1)}) p_s^{(b(s)+2)} \sum_{t\in\Gamma(s)} \prod_{v\in\Gamma(t)} \prod_{i=1}^{b(v)} (1-p_v^{(i)}). \tag{4}$$

Let $\alpha := \sum_{t\in\Gamma(s)} \prod_{v\in\Gamma(t)} \prod_{i=1}^{b(v)} (1-p_v^{(i)})$ for simplicity of notations. Then we have $f(b+\chi_s) - f(b) - (f(b+2\chi_s) - f(b+\chi_s)) = p_s^{(b(s)+1)}\alpha - p_s^{(b(s)+2)}(1-p_s^{(b(s)+1)})\alpha \geq \alpha(p_s^{(b(s)+2)} - p_s^{(b(s)+2)}(1-p_s^{(b(s)+2)})) = \alpha(p_s^{(b(s)+2)})^2 \geq 0$. $\square$

C Pseudocode of Algorithm

Algorithm 3 SIMPLEGREEDYPROCEDUREFORCOMPETITORMODEL

for each $t \in T$ **do**
  Let $\lambda := 1$
  for $k = 1$ to $1 + \max_{s\in\Gamma(t)} \tilde{b}(s)$ **do**
    Let $\phi_k(t) := 1$ and $\lambda_{t,k} := \lambda \left(1 - \prod_{s\in\Gamma(t):\tilde{b}(s)\geq k} (1-\tilde{p}_s^{(k)})\right)$.
    Let $\lambda := \lambda \prod_{s\in\Gamma(t):\tilde{b}(s)\geq k} (1-\tilde{p}_s^{(k)})$.
  end for
end for
Let $b := 0$.
for $i = 1$ to B **do**
  Choose s maximizing $\sum_{t\in\Gamma(s)} \sum_k \lambda_{t,k} r_{s,k}^{(b(s)+1)} \phi_k(t)$.
  Let $b(s) := b(s) + 1$.
  for $t \in \Gamma(s)$ **do**
    for $k = 1$ to $1 + \max_{s\in\Gamma(t)} \tilde{b}(s)$ **do**
      Let $\phi_k(t) := (1 - r_{s,k}^{(b(s))})\phi_k(t)$.
    end for
  end for
end for
return b