# Supplementary Material: Large-scale Log-determinant Computation through Stochastic Chebyshev Expansions

## A. Proof of Corollary 3

For given $\varepsilon < \frac{2}{\log(\sigma_{\max}^2)}$, set $\varepsilon_0 = \frac{\varepsilon}{2} \log\left(\frac{1}{\sigma_{\max}^2}\right)$. Since all eigenvalues of $C^T C$ are positive and less than 1, it follows that

$$\left|\log\det\left(C^T C\right)\right| = \left|\sum_{i=1}^{d} \log \lambda_i\right| \geq d \log\left(\frac{1}{\sigma_{\max}^2}\right)$$

where $\lambda_i$ are $i$-th eigenvalues of $C^T C$. Thus,

$$\varepsilon_0 = \frac{\varepsilon}{2} \log\left(\frac{1}{\sigma_{\max}^2}\right) \leq \frac{\varepsilon}{2} \frac{\left|\log\det C^T C\right|}{d} = \varepsilon \frac{\left|\log\left(\left|\det C\right|\right)\right|}{d}$$

We use $\varepsilon_0$ instead of $\varepsilon$ from Theorem 2, then following

$$\Pr\left[\ \left|\log\left(\left|\det C\right|\right) - \Gamma\right| \leq \varepsilon \left|\log\left(\left|\det C\right|\right)\right|\ \right] \geq 1 - \zeta$$

holds if $m$ and $n$ satifies below condition.

## B. Proof of Corollary 4

Similar to proof of Corollary 3, set $\varepsilon_0 = \frac{\varepsilon}{2} \log \sigma_{\min}^2$. Since eigenvalues of $C^T C$ are greater than 1,

$$\left|\log\det\left(C^T C\right)\right| \geq d \log \sigma_{\min}^2$$

and $\varepsilon_0 \leq \varepsilon \frac{\left|\log(\left|\det C\right|)\right|}{d}$. From Theorem 2, we substitute $\varepsilon_0$ into $\varepsilon$ and

$$\Pr\left[\ \left|\log\det C - \Gamma\right| \leq \varepsilon \left|\log\det C\right|\ \right] \geq 1 - \zeta$$

holds if $m$ and $n$ satifies below condition.

## C. Proof of Corollary 5

For $\varepsilon_0 = \varepsilon(\Delta_{\texttt{avg}} - 1)/2, \zeta \in (0,1)$, Theorem 2 provides the following inequality:

$$\Pr\left(\left|\log\det L(i^*) - \Gamma\right| \leq \varepsilon_0(|V| - 1)\right) \geq 1 - \zeta.$$

Observe that since vertex $i^*$ is connected all other vertices, the number of spanning tree, i.e., $\det L(i^*)$, is greater than $2^{(|V|-1)(\Delta_{\texttt{avg}}-1)/2}$. Hence, we have

$$\begin{aligned}&\Pr\left(\left|\log\det L(i^*) - \Gamma\right| \leq \varepsilon_0(|V| - 1)\right)\\ &\quad\leq \Pr\left(\left|\log\det L(i^*) - \Gamma\right| \leq \varepsilon \log\det L(i^*)\right).\end{aligned}$$

This completes the proof of Corollary 5.