Multi-view Sparse Co-clustering via Proximal Alternating Linearized Minimization

Appendix A: related definitions

Definition 1 (*Semi-algebraic sets and functions*) A subset S of $\mathbb{R}^d$ is a real semi-algebraic set if there exists a finite number of real polynomial functions g_{ij}, h_{ij}: $\mathbb{R}^d \to \mathbb{R}$ such that

$$S = \bigcup_{j=1}^{p} \bigcap_{i=1}^{q} \{\mathbf{u} \in \mathbb{R}^d : g_{ij}(\mathbf{u}) = 0 \; and \; h_{ij}(\mathbf{u}) < 0\}.$$

Moreover, a function h is called semi-algebraic if its graph

$$\{(u, t) \in \mathbb{R}^{d+1} : h(u) = t\}$$

is a semi-algebraic subset of $\mathbb{R}^{d+1}$.

Semi-algebraic sets are stable under the operations of finite union, finite intersections, complementation and Cartesian product. The following are the semi-functions or the property of semi-functions that are used in the main text:

- Indicator function of semi-algebraic sets.
- Finite sums and product of semi-algebraic functions.
- Composition of semi-algebraic functions.
- Real polynomial functions.

For any subset $S \in \mathbb{R}^d$ and any point $\mathbf{x} \in \mathbb{R}^d$, we define the distance from $\mathbf{x}$ to S as follows:

$$\text{dist}(\mathbf{x}, S) := \inf\{\|\mathbf{y} - \mathbf{x}\| : \mathbf{y} \in S\}.$$

when $S \in \emptyset$, we have dist$(\mathbf{x}, S) = 0$ fro all $\mathbf{x}$. Let $\eta \in (0, +\infty]$ and Φ_η be the class of all concave and continuous functions $\psi : [0, \eta) \to R_+$ that satisfy the following conditions: (1) $\phi(0) = 0$; (2) ψ is C^1 on $(0, \eta)$ and continuous at 0; (3) for all $s \in (0, \eta) : \psi'(s) > 0$.

Definition 2 (*Kurdyka-Lojasiewicz property*) Let $\sigma : \mathbb{R} \to (-\infty, +\infty]$ be a proper and lower semicontinuous.

(i) The function σ is said to have the *Kurdyka-Lojasiewicz (KL) property* at $\bar{\mathbf{u}} \in \text{dom}\partial\sigma := \{\mathbf{u} \in \mathbb{R}^d : \partial\sigma \neq \emptyset\}$ if there exists $\eta \in (0, +\infty]$, a neighborhood $\mathbf{U}$ of $\bar{\mathbf{u}}$ and a function $\psi \in \Phi_\eta$, such that for all

$$\mathbf{u} \in \mathbf{U} \cap [\sigma(\bar{\mathbf{u}}) < \sigma(\mathbf{u}) < \sigma(\bar{\mathbf{u}}) + \eta],$$

the following equality holds

$$\psi'(\sigma(\mathbf{u}) - \sigma(\bar{\mathbf{u}}))\text{dist}(0, \partial\sigma(\mathbf{u})) \geq 1.$$

(ii) If σ satisfy KL property at each point of dom $\partial\sigma$ then σ is called KL function.