A Proof of Theorem 1

Here, we give a full proof of **Theorem 1**. Recall that an equivalent construction of the PAGA graphs is obtained by collapsing the graph G_1^{mt} to G_m^t leading to t super nodes. Accordingly, based on G_m^t, we can directly generate G_m^{t+1} by (temporarily) adding m more nodes, each linking with a single edge, and then collapsing these m nodes to the $(t+1)$th super node. Let $\mathsf{d}_{t,i}(v)$ denote the degree of the *super-node* v after $mt+i-1$ edges have been introduced, the probability that the $mt+i$th node (or equivalently its corresponding super-node $t+1$) connects to v is

$$\Pr\{(t+1,v)\} = \begin{cases} \frac{\mathsf{d}_{t,i}(v)+m(f(v,t+1)-1)}{(\overline{f}(\cdot,t+1)+1)(mt+i)-1} & 1 \le v \le t \\ \frac{\mathsf{d}_{t,i}(t+1)+i\cdot(\overline{f}(\cdot,t+1)-1)+1}{(\overline{f}(\cdot,t+1)+1)(mt+i)-1} & v = t+1. \end{cases} \tag{15}$$

Given these definitions, we begin with proving *statement 1*. With Eq. (9) and starting conditions

$$N(k,1) = [k=2m], \quad N(k,t) = N(k,t,1) \tag{16}$$

we get a complete description of $N(k,t)$ as Eq. (8) of *statement 1*

$$N(k,t+1) = N(k,t,m+1) + \Pr(\mathsf{d}_{t,m+1}(t+1) = k)$$

Next we prove *statement 2* which is straightforward from the definition. First, note that $k \in \{m, m+1, ..., 2m\}$, i.e.:

$$\Pr(\mathsf{d}_{t,k+1}(t+1)k) = 0, \quad k < m \text{ or } k > 2m. \tag{17}$$

The minimal value $\mathsf{d}_{t,m+1}(t+1) = m$ is obtained when no one of the k edges is a loop. In this case, $\mathsf{d}_{t,i}(t+1) = i-1$ for all i, so:

$$\begin{aligned}\Pr(\mathsf{d}_{t,m+1}(t+1) = m) &= \prod_{i=1}^{m}\left(1 - \frac{i\cdot\overline{f}(\cdot,t+1)}{(\overline{f}(\cdot,t+1)+1)(mt+i)-1}\right) \\ &= 1 + O_m\left(\frac{1}{t\overline{f}(\cdot,t+1)}\right).\end{aligned}$$

From $\sum_{k=m}^{2m} \Pr(\mathsf{d}_{t,m+1} = k) = 1$ and $\Pr(\mathsf{d}_{t,m+1} = k) \ge 0$:

$$\Pr(\mathsf{d}_{t,m+1}(t+1) = k) = O_m\left(\frac{1}{t\overline{f}(\cdot,t+1)}\right), m < d \le 2m. \tag{18}$$

Finally the claim follows from the definition of $[k=m]$.

Before proving *statement 3*, we first derive some properties of $c(k,t,i)$ and prove *statement 4* en-route. Here, for brevity, we denote $c(k,t,i)$ as $c(k)$. Starting from $c(m)$ we study the step-wise change of $c(k)$ as:

$$\begin{aligned}c(m) &= \frac{B(m\cdot\overline{f}(m,t,i), \overline{f}(\cdot,t)+2)}{B(m\cdot\overline{f}(m,t,i), \overline{f}(\cdot,t)+1)} \\ &= \frac{\Gamma(\overline{f}(\cdot,t)+2)}{\Gamma(\overline{f}(\cdot,t)+1)} \times \frac{\Gamma(m\cdot\overline{f}(m,t,i)+\overline{f}(\cdot,t)+1)}{\Gamma(m\cdot\overline{f}(m,t,i)+\overline{f}(\cdot,t)+2)} \\ &= \frac{\overline{f}(\cdot,t)+1}{m\cdot\overline{f}(m,t,i)+\overline{f}(\cdot,t)+1}\end{aligned} \tag{19}$$

For $k > m$, the ratio of $c(k,i)$ to $c(k-1,i)$ can be simplified as:

$$\begin{aligned}\frac{c(k-1)}{c(k)} &= \frac{B\left(k-1+m(\overline{f}(k-1,t,i)-1)), \overline{f}(\cdot,t)+2\right)}{B\left(k+m(\overline{f}(k,t,i)-1)), \overline{f}(\cdot,t)+2\right)} \\ &= \frac{\Gamma\left(k-1+m(\overline{f}(k-1,t,i)-1)\right)}{\Gamma\left(k+m(\overline{f}(k,t,i)-1)\right)} \times \frac{\Gamma\left(k+m(\overline{f}(k,t,i)-1))+\overline{f}(\cdot,t)+2\right)}{\Gamma\left(k-1+m(\overline{f}(k-1,t,i)-1)+\overline{f}(\cdot,t)+2\right)} \\ &= \frac{k+m(\overline{f}(k,t)-1)+1+\overline{f}(\cdot,t)}{k-1+m(\overline{f}(k-1,t)-1)},\end{aligned} \tag{20}$$

where we make the approximation $\overline{f}(k,t,i) \simeq \overline{f}(k-1,t,i)$ with the assumption that the average similarity of nodes with degree k to node t is similar to that of nodes with degree $k+1$. In particular, $c(k-1) > c(k)$, so $c(k) < c(m) < 1$ for all $k \geq m$.

Now to analyze the asymptotic behavior of $c(k)$ we begin by taking logarithm as:

$$\begin{aligned} \ln c(k,i) &= C_0 + \ln\Gamma(k+C_1) - \ln\Gamma(k+C_1+\overline{f}(\cdot,t)+2) \\ C_0 &= \ln\frac{\Gamma(\overline{f}(\cdot,t)+2)}{B(m\cdot\overline{f}(m,t,i),\overline{f}(\cdot,t)+1)}, C_1 = m(\overline{f}(k,t,i)-1) \end{aligned} \tag{21}$$

Exploiting the fact that asymptotically
$\Gamma(x+1) \sim \sqrt{2\pi x}\left(\frac{x}{e}\right)^x \overset{\ln}{\to} \ln\sqrt{2\pi} + x(\ln x - 1) + \frac{1}{2}\ln x$, Eq. (21) becomes:

$$\begin{aligned} &C_0 + (k+C_1)\left(\ln(k+C_1)-1\right) + O\left(\frac{1}{k}\right) - \left(k+C_1+\overline{f}(\cdot,t)+2\right)\left(\ln(k+C_1+\overline{f}(\cdot,t)+2)-1\right) \\ =&C_0 + (k+C_1)\left(\ln k + \frac{C_1}{k} - 1\right) + O\left(\frac{1}{k}\right) - \left(k+C_1+\overline{f}(\cdot,t)+2\right)\left(\ln k + \frac{C_1+\overline{f}(\cdot,t)+2}{k} - 1\right) \\ =&C_0 - \left(2+\overline{f}(\cdot,t)\right)\ln k + O\left(\frac{1}{k}\right) \end{aligned}$$

Rewriting the C_0 term, the full expression of asymptotic value of $c(k)$ as k grows becomes

$$c(k) = \frac{\Gamma(\overline{f}(\cdot,t)+2)}{B(m(\overline{f}(m,t,i)),\overline{f}(\cdot,t)+1)}k^{-2-\overline{f}(\cdot,t)}\left(1+O\left(\frac{1}{k}\right)\right).$$

To prove the *statement 3*, we show

$$\begin{aligned} &N(k,t,i+1) \\ &= \sum_{v=1}^{t}\Pr(\mathsf{d}_{t,i+1}(v)=k) \\ &= \sum_{v=1}^{t}(\Pr(\mathsf{d}_{t,i+1}(v)=k,\gamma=v) + \Pr(\mathsf{d}_{t,i+1}(v)=k,\gamma\neq v)) \\ &= \sum_{v=1}^{t}(\Pr(\mathsf{d}_{t,i}(v)=k-1,\gamma=v) + \Pr(\mathsf{d}_{t,i}(v)=k,\gamma\neq v)) \\ &= \sum_{v=1}^{t}\left(\Pr(\mathsf{d}_{t,i}(v)=k-1)\frac{k-1+m(f(v,t+1)-1)}{(\overline{f}(\cdot,t+1)+1)(mt+i)-1} + \Pr(\mathsf{d}_{t,i}(v)=k)\left(1-\frac{k+m(f(v,t+1)-1)}{(\overline{f}(\cdot,t+1)+1)(mt+i)-1}\right)\right) \\ &= N(k-1,t,i)\frac{k-1+m(\overline{f}(k-1,t+1,i)-1)}{(\overline{f}(\cdot,t+1)+1)\cdot(mt+i)-1} + N(k,t,i)\left(1-\frac{k+m(\overline{f}(k,t+1,i)-1)}{(\overline{f}(\cdot,t+1)+1)\cdot(mt+i)-1}\right) \end{aligned} \tag{22}$$

By proving *statement 4*, we complete our proof of **Theorem 1**. For the ease of proof, let us define $\tilde{N}(k,t,i)$:

$$\tilde{N}(k,t,i) = N(k,t,i) - c(k,t,i)\left(t + \frac{i}{m} - \frac{1}{m(1+\overline{f}(\cdot,t))}\right) \tag{23}$$

The lhs of (11) can be expressed in a recursive fashion as:

$$\begin{aligned} &\tilde{N}(k,t,i+1) + \frac{i}{m}[k=m] \\ &\quad= \tilde{N}(k,t,i+1) + \frac{1}{m}[k=m] + \frac{i-1}{m}[k=m] \\ &\quad= \tilde{N}(k,t,i+1) + [k=m]c(m,t,i)\frac{m(\overline{f}(m,t,i))+\overline{f}(\cdot,t)+1}{m(1+\overline{f}(\cdot,t))} + \frac{i-1}{m}[k=m] \end{aligned}$$

$$= N(k,t,i+1) - c(k,t,i)\frac{(\overline{f}(\cdot,t)+1)(mt+i+1)-1}{m(1+\overline{f}(\cdot,t))} - c(m,t,i)\frac{[k=m](m(\overline{f}(m,t,i)) + \overline{f}(\cdot,t)+1)}{m(1+\overline{f}(\cdot,t))} + \frac{i-1}{m}[k=m]$$

$$= N(k,t,i+1) - \left(\frac{(\overline{f}(\cdot,t)+1)(mt+i)-1}{m(\overline{f}(\cdot,t)+1)}\right)$$

$$\times\, c(k,t,i)\frac{(\overline{f}(\cdot,t)+1)(mt+i+1)-1-[k=m]\left(k+m(\overline{f}(k,t)-1)+\overline{f}(\cdot,t)+1\right)}{(1+\overline{f}(\cdot,t))\cdot(mt+i)-1} + \frac{i-1}{m}[k=m]$$

$$= N(k,t,i+1) + \frac{i-1}{m}[k=m] - \left(t+\frac{i}{m}-\frac{1}{m(1+\overline{f}(\cdot,t))}\right)\frac{c(k,i)}{(1+\overline{f}(\cdot,t))(mt+i)-1}$$

$$\times \left\{(1-[k=m])\left(k+m(\overline{f}(k,t,i)-1)+1+\overline{f}(\cdot,t)\right) + \left((\overline{f}(\cdot,t)+1)(mt+i)-1\right) - \left(k+m(\overline{f}(k,t,i)-1)\right)\right\}$$

$$= N(k,t,i+1) + \frac{i-1}{m}[k=m] - \left(t+\frac{i}{m}-\frac{1}{m(1+\overline{f}(\cdot,t)}\right)$$

$$\times\left(c(k-1,t,i)\frac{k-1+m(\overline{f}(k-1,t,i)-1)}{(1+\overline{f}(\cdot,t))\cdot(mt+i)-1} + c(k,i)\left(1-\frac{k+m(\overline{f}(k,t,i)-1)}{(1+\overline{f}(\cdot,t))(mt+i)-1}\right)\right)$$

$$= \tilde{N}(k-1,t,i)\frac{k-1+m(\overline{f}(k-1,t,i)-1)}{(1+\overline{f}(\cdot,t))\cdot(mt+i)-1} + \tilde{N}(k,t,i)\left(1-\frac{k+m(\overline{f}(k,t,i)-1)}{(1+\overline{f}(\cdot,t))(mt+i)-1}\right) + \frac{i-1}{m}[k=m]$$

Note that the last line comes from Eq. (22) and Eq. (23). Now, in a fashion similar to [18] involving manipulations like the ones done above, we have $\tilde{N}(k,t,i) + [k=m]\frac{i-1}{m} = O_m\left(\frac{1}{k}\right)$ finishing the proof of the theorem.

B Proof of Theorem 3

The proof of the bound on the diameter for PAGA graphs can by obtained by extending the original proof for the standard preferential attachment model from a uniform measure to a non-uniform measure, described by the affinity function f. Here, we ignore the case of having disconnected components in a graph as the graph will be one connected component with high probability.[2] As self loops do not affect the eccentricity of a node and hence the diameter, we simply ignore them in the generation process itself – for the purpose of bounding the diameter.

We begin the proof by noting that for $m = 1$ the graph is a tree and for the cases $m \geq 2$, which are formed by collapsing the graph G_1^{mt} graph, the diameter can only shrink. Next, the diameter of the graph can not be larger than twice the height of this tree, which is equal to the maximal graph distance between vertex 1 and any of the other vertices. So, it is sufficient to bound the height of the tree.

For bounding the tree height, we follow the steps of [48] and outline here the differences.[3] We start with a continuous time branching process, where the rate is given by $\lambda^t(j) = \mathsf{d}(j) + f(j,t)$. Therefore, the overall transition rate after t vertices are present (i.e. after $t-1$ births) is given by:

$$\sum_{j=1}^{t}(\mathsf{d}(j) + f(t,j) = 2t + t\bar{f}(\cdot,t+1) \tag{24}$$

Now we can decompose the time τ_t as a sum of independent variables, exponentially distributed with parameter $2t + t\bar{f}(\cdot,t+1)$, i.e.

$$\tau_t = \sum_{j=1}^{t}\mathsf{t}_j$$

where $\mathsf{t}_j \sim \mathrm{Exp}(2j + j\bar{f}(\cdot,j+1))$. It follows that the mean and variance are bounded by

$$\mathbf{E}[\tau_t] = \sum_{j=1}^{t}\frac{1}{2j + j\bar{f}(\cdot,j+1)} = O(\log t)$$

[2]In order for a PAGA graph to have a disconnected component, a new super-node $t+1$ has to make m self loops. This probability asymptotically goes to 0. Note that even in the uniform case this probability is $\frac{1}{t}^m$, while the self-loop probability in PAGA is typically smaller than $1/t$.

[3]To avoid clash of notations, we also redefine the birth times as $\tau_1, .., \tau_t$ instead of t_i.

$$\mathrm{var}(\tau_t) = \sum_{j=1}^{t} \frac{1}{2j + j\bar{f}(\cdot, j+1)} = O(1)$$

These conditions match the ones required in [48]. Accordingly, the small world property holds for the PAGA model.

C Proof of Theorem 4

Note that at every timestep, there can be at most $\frac{m(m-1)}{2}$ new triangles added. So the number of triangles $T(G_m^t)$ is bounded by $O(n)$. On the other hand, the number of triplets in G_m^t follows $\sum_{k=1}^{d_{max}} N(k,t)\binom{k}{2} \propto \sum_{k=1}^{d_{max}} t \cdot k^{\bar{f}(\cdot,t)}$. As the sum $\sum_{k=1}^{d_{max}} k = mt$, it is straightforward that $C(G_m^t) \to 0$ unless $\bar{f}(\cdot, t) > 1$. We omit the analysis for the cases where $\bar{f}(\cdot, t) > 1$ as real graphs follow power law exponent around -2.1 to -2.5 ($0.1 \leq \bar{f}(\cdot, t) \leq 0.5$) □

D Spyplot of W

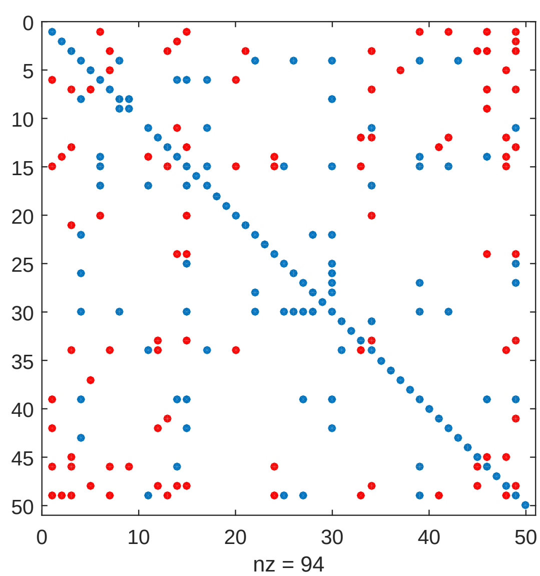

Figure 6: Spyplot of the matrix W. Blue entries correspond to positive elements; red entries to negative elements

E Topics and keywords learned by LDA

We used 50 topics (i.e. W is a 50×50 matrix) for learning the affinity function f. Here, we represent the top 20 keywords for top 10 topics we obtained from LDA.

Topic 0	Topic 1	Topic 2	Topic 3	Topic 4
method	quantization	gauge	theory	gauge
integral	hamiltonian	invariant	field	magnetic
path	dirac	invariance	approach	nonabelian
using	zero	local	based	abelian
functional	modes	brst	discuss	chernsimons
regularization	canonical	lagrangian	nonperturbative	electric
approach	physical	covariant	presented	charge
formula	lightcone	constraints	properties	theory
use	coordinates	formulation	new	monopole
integrals	shown	formalism	methods	su
expansion	quantized	class	may	monopoles
used	formalism	first	present	flux
series	variables	ghost	talk	charged
procedure	mode	cohomology	version	higgs
technique	constraint	extended	interpretation	dual
applied	leads	fields	developed	term
obtained	operator	lorentz	techniques	vortex
integration	approach	fixing	provide	selfdual
expression	constraints	transformations	used	vortices
formalism	formulation	auxiliary	analysis	yangmills
Topic 5	**Topic 6**	**Topic 7**	**Topic 8**	**Topic 9**
n	matrix	coupling	loop	form
supersymmetric	model	point	theory	general
supersymmetry	matrices	limit	perturbation	parameters
supergravity	ansatz	fixed	renormalization	arbitrary
superconformal	spectral	large	wilson	given
super	models	strong	finite	explicit
superspace	relation	flow	perturbative	parameter
yangmills	limit	points	expansion	q
multiplet	integrable	constant	one	values
superfield	spin	group	order	set
multiplets	smatrix	renormalization	loops	independent
cal	scattering	weak	non	can
bps	elements	theory	result	terms
theories	eigenvalues	infrared	lambda	one
harmonic	chain	constants	cutoff	structure
susy	bethe	rg	diagrams	forms
superfields	corresponding	behavior	divergences	present
supersymmetries	factors	view	expectation	real
sym	related	critical	values	allows
large	representation	expansion	scheme	showns

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