The entropic barrier: a simple and optimal universal self-concordant barrier

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Abstract
We prove that the Fenchel dual of the log-Laplace transform of the uniform measure on a convex body in $\mathbb{R}^n$ is a $(1 + o(1))n$-self-concordant barrier, improving a seminal result of Nesterov and Nemirovski. This gives the first explicit construction of a universal barrier for convex bodies with optimal self-concordance parameter. The proof is based on basic geometry of log-concave distributions, and elementary duality in exponential families. The result also gives a new perspective on the minimax regret for the linear bandit problem.