Appendix to Hierarchical Compound Poisson Factorization

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1. Proofs

1.1. Proof of Theorem 2

Proof. Follows directly from Theorem 1. □

1.2. Proof of Remark 1

Proof. Let $M_X(t)$ be the moment generating function (MGF) of $X \sim p_\Psi(x; \theta, \kappa)$, then MGF of X_+ is given by $M_{X_+}(t) = e^{\Lambda(M_X(t)-1)}$. MGF of an ordinary Poisson random variable Y with parameter Λ is $M_Y(t) = e^{\Lambda(e^t-1)}$. If $M_{X_+}(t) = M_Y(t)$, then $M_X(t) = e^t$ which is the MGF for degenerate distribution δ_1. If $M_X(t) = e^t$, then $M_{X_+}(t) = M_Y(t)$. □

1.3. Proof of Theorem 3

Proof. Let $M_X(t)$ be the MGF of $X \sim p_\Psi(x; \theta, \kappa)$ and $\left\{X_{++}^m\right\}_{m=1}^{\infty}$ be a sequence of random variables where $X_{++}^m = X_+^m \mid X_+^m \neq 0$ and $X_+^m \sim p_\Psi(x; \theta, \kappa, \Lambda = \frac{1}{m})$. The MGF of X_{++}^m is given by

$$M_{X_{++}^m}(t) = \frac{e^{M_X(t)/m} - 1}{e^{1/m} - 1}.$$

Since $\lim_{m\to\infty} M_{X_{++}^m}(t) = M_X(t)$, X_{++} converges to X in distribution as Λ goes to zero. □