Supplementary Material

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1 Sketch of the Proof of Lemma 2

Proof. We describe a sketch of the proof. Note that w_j^2 is subject to a Gamma distribution $\mathrm{Ga}(n/2, 2\eta/n)$ with $\eta = w_j^{*2}$. It forms an exponential family, of which w_j^2 is a sufficient statistic and η is the expectation parameter. Noting that the KL-divergence from $\eta = (1\pm\epsilon)w_j^{*2}$ to $\eta = w_j^{*2}$ is $(n/2)(\pm\epsilon - \log(1\pm\epsilon))$, we can prove (8) by using a large deviation inequality shown in Csiszar (1984). The next inequality follows from $(1-t)^p \geq 1-pt$ for any $t \in [0,1]$ and $p \geq 1$. To simplify the bound, we can do more. The maximum positive real number a such that, for any $\epsilon \in [0,1]$, $a\epsilon^2 \leq (1/2)(\epsilon - \log(1+\epsilon))$ is $(1-\log 2)/2$. Then, the maximum integer z such that $(1-\log 2)/2 \geq 1/z$ is 7, which gives the last inequality. □

1.1 Proof of Theorem 6

It is not necessary to start from scratch. We reuse the proof of Theorem 5 in the manuscript.

Proof. We can start from (13) in the manuscript. For convenience, we define

$$\begin{aligned}\xi(x^n, y^n) &= \frac{1}{n}\max_{\theta\in\Theta}\left\{F_\lambda^\theta(x^n, y^n) - L(\theta|x^n)\right\} \\ &= \max_{\theta\in\widetilde{\Theta}}\left\{\frac{d_\lambda^n(p_*, p_\theta)}{n} - \frac{1}{n}\log\frac{p_*(y^n|x^n)}{p_\theta(y^n|x^n)} - \frac{L(\theta|x^n)}{n}\right\}.\end{aligned}$$

By Markov's inequality and (13),

$$\begin{aligned}\Pr\left(\xi(x^n, y^n) \geq \tau | x^n \in A_\epsilon^n\right) &= \Pr\left(\exp\left(\frac{n\xi(x^n, y^n)}{\beta}\right) \geq \exp\left(\frac{n\tau}{\beta}\right) \middle| x^n \in A_\epsilon^n\right) \\ &\leq \frac{\exp\left(-\frac{n\tau}{\beta}\right)}{P_\epsilon^n}.\end{aligned}$$

Hence, we obtain

$$\begin{aligned}\Pr\left(\xi(x^n, y^n) \geq \tau\right) &= P_\epsilon^n \Pr\left(\xi(x^n, y^n) \geq \tau | x^n \in A_\epsilon^n\right) + (1 - P_\epsilon^n)\Pr\left(\xi(x^n, y^n) \geq \tau | x^n \notin A_\epsilon^n\right) \\ &\leq P_\epsilon^n \Pr\left(\xi(x^n, y^n) \geq \tau | x^n \in A_\epsilon^n\right) + (1 - P_\epsilon^n) \\ &\leq \exp(-n\tau/\beta) + (1 - P_\epsilon^n).\end{aligned}$$

The proof completes by noticing that $(1/n)\left(F_\lambda^{\hat{\theta}}(x^n, y^n) - L(\hat{\theta}|x^n)\right) \leq \xi(x^n, y^n)$ for any x^n and y^n. □

References

Csiszar, I. Sanov property, generalized I-projection and a conditional limit theorem. *The Annals of Probability*, 12:768–793, 1984.