1 Sketch of the Proof of Lemma 2

Proof. We describe a sketch of the proof. Note that $w_j^2$ is subject to a Gamma distribution $\text{Ga}(n=2, 2\frac{n}{n})$ with $\eta = w_j^2$. It forms an exponential family, of which $w_j^2$ is a sufficient statistic and $\eta$ is the expectation parameter. Noting that the KL-divergence from $\eta = (1+\epsilon)w_j^2$ to $\eta = w_j^2$ is $(n/2)(\epsilon - \log(1+\epsilon))$, we can prove (8) by using a large deviation inequality shown in Csiszar (1984). The next inequality follows from $(1-t)p \geq 1-pt$ for any $t \in [0, 1]$ and $p \geq 1$. To simplify the bound, we can do more. The maximum positive real number $a$ such that, for any $\epsilon \in [0, 1]$, $ae^2 \leq (1/2)(\epsilon - \log(1+\epsilon))$ is $(1 - \log 2)/2$. Then, the maximum integer $z$ such that $(1 - \log 2)/2 \geq 1/z$ is 7, which gives the last inequality. □

1.1 Proof of Theorem 6

It is not necessary to start from scratch. We reuse the proof of Theorem 5 in the manuscript.

Proof. We can start from (13) in the manuscript. For convenience, we define

$$
\xi(x^n, y^n) = \frac{1}{n} \max_{\theta \in \Theta} \left\{ F_\lambda(x^n, y^n) - L(\theta|x^n) \right\}
$$

$$
= \max_{\theta \in \Theta} \left\{ \frac{d^n(p*)}{n} - \frac{1}{n} \log \frac{p_*(y^n|x^n)}{p(y^n|x^n)} - \frac{L(\theta|x^n)}{n} \right\}.
$$

By Markov’s inequality and (13),

$$
\Pr(\xi(x^n, y^n) \geq \tau|x^n \in A^n) = \Pr \left( \exp \left( \frac{n\xi(x^n, y^n)}{\beta} \right) \geq \exp \left( \frac{n\tau}{\beta} \right) \bigg| x^n \in A^n \right) 
$$

$$
\leq \frac{\exp \left( - \frac{n\tau}{\beta} \right)}{P^n}. 
$$

Hence, we obtain

$$
\Pr(\xi(x^n, y^n) \geq \tau) = P^n \Pr(\xi(x^n, y^n) \geq \tau|x^n \in A^n) + (1 - P^n) \Pr(\xi(x^n, y^n) \geq \tau|x^n \notin A^n)
$$

$$
\leq P^n \Pr(\xi(x^n, y^n) \geq \tau|x^n \in A^n) + (1 - P^n)
$$

$$
\leq \exp(-n\tau/\beta) + (1 - P^n).
$$

The proof completes by noticing that $(1/n) \left( F_\lambda(x^n, y^n) - L(\hat{\theta}|x^n) \right) \leq \xi(x^n, y^n)$ for any $x^n$ and $y^n$. □

References