Supplementary Material for the paper "Controlling the distance to a Kemeny consensus without computing it"

1. Examples of voting rules

We elaborate in detail the voting rules used in the paper to approximate Kemeny's rule. Note that if multiple consensuses are returned from a rule on a given dataset, we randomly pick one from these consensuses.

Positional scoring rules. Given a scoring vector $w = (w_1, ..., w_n) \in \mathbb{R}^n$ of weights respectively for each alternative in $[\![n]\!]$, the ith alternative in a vote scores w_i. A full ranking is given by sorting the averaged scores over all votes, for example, the winner is the alternative with highest total score over all the votes. The **plurality** rule has the weight vector $(1, 0, ..., 0)$, the k**-approval** rule has $(1, ..., 1, 0..., 0)$ containing 1s in the first k positions, and the **Borda** rule has $(n, n-1, ..., 1)$.

Copeland. A full ranking is given by sorting the Copeland scores averaged over all votes, for which the score of alternative i is $\sum_{j \neq i} \text{beats}(i, j)$. For example, the Copeland winner is the alternative that wins the most pairwise elections.

QuickSort. (Ali & Meilă, 2012) QuickSort recursively divides an unsorted list into two lists – one list comprising alternatives that occur before a chosen index (called the *pivot*), and another comprising alternatives that occur after, and then sorts each of the two lists. The pivot is always chosen as the first alternative.

Pick-a-Perm. (Ali & Meilă, 2012) A full ranking is picked randomly from $\mathfrak{S}_n$ according to the empirical distribution of the dataset $\mathcal{D}_N$.

Plackett-Luce. A Plackett-Luce ranking model defined for any $\sigma \in \mathfrak{S}_n$ by $p_w(\sigma) = \prod_{i=1}^{n} w_{\sigma(i)} / \left(\sum_{j=i}^{n} w_{\sigma(j)} \right)$ parameterized by $w = (w_1, \ldots, w_n) \in \mathbb{R}^n$, fitted to $\mathcal{D}_N$ by means of the MM algorithm (Hunter, 2004). A full ranking is then given by sorting w.

Pick-a-Random. A full ranking is picked randomly from $\mathfrak{S}_n$ according to uniform law (independent from $\mathcal{D}_N$). Qualitatively speaking Pick-a-Random is expected as a negative control experiment. To intuitively understand the rationale behind Pick-a-Random, let us consider the case conditioned on that the output of a voting rule has (at least) certain Kendall's tau distance to the Kemeny consensus. Compared to what Pick-a-Random would blindly pick any permutation without accessing to the dataset $\mathcal{D}_N$ at all, a sensible voting rule should have a better chance to output one permutation with a smaller angle θ with $\phi(\mathcal{D}_N)$ among all the permutations that share the same distance to Kemeny consensus. As we have reasoned in the geometric proof of the method that the smaller the angle θ is, the more applicable our method will be, Pick-a-Random is expected to perform worse than other voting rules in terms of applicability of our method.

References

Ali, A. and Meilă, M. Experiments with kemeny ranking: What works when? *Mathematical Social Sciences*, 64 (1):28–40, 2012.

Hunter, D. R. MM algorithms for generalized bradley-terry models. *Annals of Statistics*, pp. 384–406, 2004.