A Proof of Lemma 2

Recall

$$Q_i^+ = \sum_{j=1}^{M} \log\left(1 + sgn(y_j)sgn(u_{ij})e^{-(K-1)w_{ij}}\right) = \sum_{j=1}^{M} \log\left(1 + sgn\left(y_j/s_{ij}\right)e^{-(K-1)w_{ij}}\right)$$

where $\frac{y_j}{s_{ij}} = x_i + \frac{\sum_{t\neq i} x_i s_{tj}}{s_{ij}} = x_i + \theta_i \frac{S_j}{s_{ij}}$. Here, $S_j \sim S(\alpha,1)$ is independent of s_{ij}, and for convenience we define $\theta = \left(\sum_{i=1}^{N} |x_i|^\alpha\right)^{1/\alpha}$ and $\theta_i = (\theta^\alpha - |x_i|^\alpha)^{1/\alpha}$. In particular, if $x_i = 0$, then $\theta_i = \theta$ and $sgn\left(y_j/s_{ij}\right) = sgn(S_j/s_{ij})$. As S_j and s_{ij} are symmetric and independent, we can replace $sgn(S_j/s_{ij})$ by $sgn(s_{ij}) = sgn(u_{ij})$. To see this

$$\begin{aligned}
&\mathbf{Pr}\left(sgn(S_j/s_{ij}) = 1\right) = \mathbf{Pr}\left(sgn(s_{ij}/S_j) = 1\right)\\
=&\mathbf{Pr}\left(sgn(s_{ij}) = 1\right)\mathbf{Pr}\left(S_j > 0\right) + \mathbf{Pr}\left(sgn(s_{ij}) = -1\right)\mathbf{Pr}\left(S_j < 0\right)\\
=&\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2} = \frac{1}{2} = \mathbf{Pr}\left(sgn(s_{ij}) = 1\right)
\end{aligned}$$

Thus, we have

$$\begin{aligned}
&\mathbf{Pr}\left(Q_i^+ > \epsilon M/K, x_i = 0\right)\\
=&\mathbf{Pr}\left(\sum_{j=1}^{M} \log\left(1 + sgn(y_j/s_{ij})\exp\left(-(K-1)w_{ij}\right)\right) > \epsilon M/K, x_i = 0\right)\\
=&\mathbf{Pr}\left(\sum_{j=1}^{M} \log\left(1 + sgn(S_j/s_{ij})\exp\left(-(K-1)w_{ij}\right)\right) > \epsilon M/K\right)\\
=&\mathbf{Pr}\left(\sum_{j=1}^{M} \log\left(1 + sgn(u_{ij})\exp\left(-(K-1)w_{ij}\right)\right) > \epsilon M/K\right)\\
=&\mathbf{Pr}\left(\prod_{j=1}^{M} \left(1 + sgn(u_{ij})\exp\left(-(K-1)w_{ij}\right)\right) > e^{\epsilon M/K}\right)\\
\leq&e^{-\epsilon M/Kt}E^M\left(1 + sgn(u_{ij})\exp\left(-(K-1)w_{ij}\right)\right)^t, \qquad (t \geq 0, \text{Markov's Inequality})\\
=&e^{-\epsilon M/Kt}\left(\frac{1}{2}E\left\{\left(1 + e^{-(K-1)w_{ij}}\right)^t + \left(1 - e^{-(K-1)w_{ij}}\right)^t\right\}\right)^M\\
=&e^{-\epsilon M/Kt}\left(\frac{1}{2}\int_0^\infty \left\{\left(1 + e^{-(K-1)w}\right)^t + \left(1 - e^{-(K-1)w}\right)^t\right\}e^{-w}dw\right)^M
\end{aligned}$$

Then we need to choose the t to minimize the upper bound. Let $b = K-1$, then

$$\begin{aligned}
&\int_0^\infty \left(1 + e^{-bw}\right)^t e^{-w}dw = \int_0^1 (1+u^b)^t du\\
=&\int_0^1 1 + u^b t + u^{2b}t(t-1)/2! + u^{3b}t(t-1)(t-2)/3! + u^{4b}t(t-1)(t-2)(t-3)/4! + ...du\\
=&1 + \frac{t}{b+1} + \frac{t(t-1)}{(2b+1)2!} + \frac{t(t-1)(t-2)}{(3b+1)3!} + ...
\end{aligned}$$

$$
\begin{aligned}
&\int_0^\infty \left(1-e^{-bw}\right)^t e^{-w} dw = \int_0^1 (1-u^b)^t du \\
=&\int_0^1 1 - u^b t + u^{2b} t(t-1)/2! - u^{3b} t(t-1)(t-2)/3! + u^{4b} t(t-1)(t-2)(t-3)/4! + ...du \\
=&1 - \frac{t}{b+1} + \frac{t(t-1)}{(2b+1)2!} - \frac{t(t-1)(t-2)}{(3b+1)3!} + ...
\end{aligned}
$$

$$
\int_0^\infty \left(1-e^{-(K-1)w}\right)^t e^{-w} + \left(1+e^{-(K-1)w}\right)^t e^{-w} dw = 2 + 2\frac{t(t-1)}{(2K-1)2!} + 2\frac{t(t-1)(t-2)(t-3)}{(4K-3)4!} + ...
$$

Therefore, for any $t \geq 0$, we have

$$
\begin{aligned}
&\mathbf{Pr}\left(Q_i^+ > \epsilon M/K, x_i = 0\right) = \mathbf{Pr}\left(Q_i^- > \epsilon M/K, x_i = 0\right) \\
\leq& e^{-\epsilon M/K t}\left(1 + \frac{t(t-1)}{(2K-1)2!} + \frac{t(t-1)(t-2)(t-3)}{(4K-3)4!} + ...\right)^M \\
=& \exp\left\{-\frac{M}{K}\left(\epsilon t - K\log\left(1 + \frac{t(t-1)}{(2K-1)2!} + \frac{t(t-1)(t-2)(t-3)}{(4K-3)4!} + ...\right)\right)\right\} \\
=& \exp\left\{-\frac{M}{K} H_1(t;\epsilon,K)\right\}
\end{aligned}
$$

where

$$
\begin{aligned}
H_1(t;\epsilon,K) &= \epsilon t - K\log\left(1 + \frac{t(t-1)}{(2K-1)2!} + \frac{t(t-1)(t-2)(t-3)}{(4K-3)4!} + ...\right) \\
H_1(t;\epsilon,\infty) &= \epsilon t - \left(\frac{t(t-1)}{2\times 2!} + \frac{t(t-1)(t-2)(t-3)}{4\times 4!} + ...\right)
\end{aligned}
$$

Note that, by L'Hospital's Rule, we have

$$
\begin{aligned}
&\lim_{K\to\infty} \frac{\log\left(1 + \frac{t(t-1)}{(2K-1)2!} + \frac{t(t-1)(t-2)(t-3)}{(4K-3)4!} + ...\right)}{1/K} \\
=& \lim_{K\to\infty} \frac{\frac{-2\frac{t(t-1)}{(2K-1)^2 2!} - 4\frac{t(t-1)(t-2)(t-3)}{(4K-3)^2 4!} + ...}{1 + \frac{t(t-1)}{(2K-1)2!} + \frac{t(t-1)(t-2)(t-3)}{(4K-3)4!} + ...}}{-1/K^2} = \frac{t(t-1)}{2\times 2!} + \frac{t(t-1)(t-2)(t-3)}{4\times 4!} + ...
\end{aligned}
$$

This completes the proof.

B Proof of Lemma 3

$$\begin{aligned}
&\mathbf{Pr}\left(Q_i^+ < \epsilon M/K, x_i > 0\right)\\
=&\mathbf{Pr}\left(\sum_{j=1}^{M}\log\left(1+sgn(y_j/s_{ij})\exp\left(-(K-1)w_{ij}\right)\right) < \epsilon M/K, x_i > 0\right)\\
=&\mathbf{Pr}\left(\exp\left(-t\sum_{j=1}^{M}\log\left(1+sgn(y_j/s_{ij})\exp\left(-(K-1)w_{ij}\right)\right)\right) > \exp\left(-t\epsilon M/K\right), x_i > 0\right),\quad t>0\\
=&\mathbf{Pr}\left(\prod_{j=1}^{M}\left(1+sgn(y_j/s_{ij})\exp\left(-(K-1)w_{ij}\right)\right)^{-t} > \exp\left(-t\epsilon M/K\right), x_i > 0\right)\\
\le&\exp\left(t\epsilon M/K\right)E^M\left(\left(1+sgn(y_j/s_{ij})\exp\left(-(K-1)w_{ij}\right)\right)^{-t}; x_i > 0\right)
\end{aligned}$$

Consider, for convenience, $\alpha \to 0$ and $x_i > 0$. Again, we study $sgn(y_j/s_{ij}) = sgn\left(x_i + \theta_i S_j/s_{ij}\right)$, where $S_j, s_{ij} \sim S(\alpha, 1)$ i.i.d. Let $T_{ij} = sgn(y_j/s_{ij})\exp\left(-(K-1)w_{ij}\right)$. As $\alpha \to 0$

$$\begin{aligned}
T_{ij} =& sgn\left(x_i + \theta_i sgn(U_j)sgn(u_{ij})\left(\frac{w_{ij}}{W_j}\right)^{1/\alpha}\right)e^{-(K-1)w_{ij}}\\
=& sgn\left(x_i + sgn(U_j)sgn(u_{ij})\left((K-1)\frac{w_{ij}}{W_j}\right)^{1/\alpha}\right)e^{-(K-1)w_{ij}}\\
=&\begin{cases} sgn(x_i)e^{-(K-1)w_{ij}} & \text{if } (K-1)w_{ij} < W_j\\ sgn(u_{ij})e^{-(K-1)w_{ij}} & \text{if } (K-1)w_{ij} > W_j\end{cases}
\end{aligned}$$

Thus,

$$\begin{aligned}
&E\left(\left(1+sgn(y_j/s_{ij})\exp\left(-(K-1)w_{ij}\right)\right)^{-t}; x_i > 0\right)\\
=&E\left\{\int_0^{W_j/(K-1)}\left(1+\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\} + \frac{1}{2}E\left\{\int_{W_j/(K-1)}^{\infty}\left(1+\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\}\\
+&\frac{1}{2}E\left\{\int_{W_j/(K-1)}^{\infty}\left(1-\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\}\\
=&\frac{1}{2}\left\{\int_0^{\infty}\left(1+\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\} + \frac{1}{2}\left\{\int_0^{\infty}\left(1-\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\}\\
+&\frac{1}{2}E\left\{\int_0^{W_j/(K-1)}\left(1+\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\} - \frac{1}{2}E\left\{\int_0^{W_j/(K-1)}\left(1-\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\}\\
=&\frac{1}{2}\int_0^1\left(1+u^b\right)^{-t}du + \frac{1}{2}\int_0^1\left(1-u^b\right)^{-t}e^{-u}du - \frac{1}{2}\int_0^{\infty}e^{-w}\int_{w/b}^1\left[\left(1-u^b\right)^{-t} - \left(1+u^b\right)^{-t}\right]dudw
\end{aligned}$$

Again, for convenience, we denote $b = K-1$.

$$\begin{aligned}
&\int_0^1(1+u^b)^{-t}du\\
=&\int_0^1 1 - u^b t + u^{2b}(-t)(-t-1)/2! + u^{3b}(-t)(-t-1)(-t-2)/3! + u^{4b}(-t)(-t-1)(-t-2)(-t-3)/4! + ...du\\
=&1 - \frac{t}{b+1} + \frac{t(t+1)}{(2b+1)2!} - \frac{t(t+1)(t+2)}{(3b+1)3!} + \frac{t(t+1)(t+2)(t+3)}{(4b+1)4!}...
\end{aligned}$$

$$\frac{1}{2}\int_0^1 (1+u^b)^{-t}du + \frac{1}{2}\int_0^1 (1-u^b)^{-t}du = 1 + \frac{t(t+1)}{(2b+1)2!} + \frac{t(t+1)(t+2)(t+3)}{(4b+1)4!} + ...$$

For the other term, we have

$$\begin{aligned}
&\frac{1}{2}\int_0^\infty e^{-w}\int_{w/b}^1 \left[\left(1-u^b\right)^{-t} - \left(1+u^b\right)^{-t}\right] dudw \\
=&\int_0^\infty e^{-w}\int_{e^{-w/b}}^1 \left[tu^b + t(t+1)(t+2)u^{3b}/3! + t(t+1)(t+2)(t+3)(t+4)u^{5b}/5! + ..\right] dudw \\
=&\left[\frac{t}{b+1} + \frac{t(t+1)(t+2)}{(3b+1)3!} + \frac{t(t+1)(t+2)(t+3)(t+4)}{(5b+1)5!} + ...\right] \\
-&\int_0^\infty e^{-w}\left[\frac{t}{b+1}(e^{-w/b})^{b+1} + \frac{t(t+1)(t+2)}{(3b+1)3!}(e^{-w/b})^{3b+1} + \frac{t(t+1)(t+2)(t+3)(t+4)}{(5b+1)5!}(e^{-w/b})^{5b+1} + ...\right] dw \\
=&\left[\frac{t}{b+1} + \frac{t(t+1)(t+2)}{(3b+1)3!} + \frac{t(t+1)(t+2)(t+3)(t+4)}{(5b+1)5!} + ...\right] \\
-&\left[\frac{t}{b+1}\frac{b}{2b+1} + \frac{t(t+1)(t+2)}{3!(3b+1)}\frac{b}{4b+1} + \frac{t(t+1)(t+2)(t+3)(t+4)}{5!(5b+1)}\frac{b}{6b+1} + ...\right]
\end{aligned}$$

Combining the results yields

$$\begin{aligned}
&E\left((1+sgn(y_j/s_{ij})\exp\left(-(K-1)w_{ij}\right))^{-t}; x_i > 0\right) \\
=&1 - \frac{t}{b+1} + \frac{t(t+1)}{(2b+1)2!} - \frac{t(t+1)(t+2)}{(3b+1)3!} + \frac{t(t+1)(t+2)(t+3)}{(4b+1)4!} - \frac{t(t+1)(t+2)(t+3)(t+4)}{(5b+1)5!} + ... \\
+&\left[\frac{t}{b+1}\frac{b}{2b+1} + \frac{t(t+1)(t+2)}{3!(3b+1)}\frac{b}{4b+1} + \frac{t(t+1)(t+2)(t+3)(t+4)}{5!(5b+1)}\frac{b}{6b+1} + ...\right] \\
=&1 - \frac{t}{2b+1} + \frac{t(t+1)}{(2b+1)2!} - \frac{t(t+1)(t+2)}{(4b+1)3!} + \frac{t(t+1)(t+2)(t+3)}{(4b+1)4!} - \frac{t(t+1)(t+2)(t+3)(t+4)}{(6b+1)5!} + ...
\end{aligned}$$

Therefore, we can write

$$\mathbf{Pr}\left(Q_i^+ < \epsilon M/K, x_i > 0\right) \leq \exp\left(-\frac{M}{K}H_2(t;\epsilon,K)\right)$$

where

$$H_2(t;\epsilon,K) = -\epsilon t - K\log\left[1 + \sum_{n=2,4,6...}^\infty \frac{1}{n(K-1)+1}\prod_{l=0}^{n-1}\frac{t+l}{n-l} - \sum_{n=1,3,5...}^\infty \frac{1}{(n+1)(K-1)+1}\prod_{l=0}^{n-1}\frac{t+l}{n-l}\right]$$

$$H_2(t;\epsilon,\infty) = -\epsilon t - \left[\sum_{n=2,4,6...}^\infty \frac{1}{n}\prod_{l=0}^{n-1}\frac{t+l}{n-l} - \sum_{n=1,3,5...}^\infty \frac{1}{(n+1)}\prod_{l=0}^{n-1}\frac{t+l}{n-l}\right]$$

C Proof of Lemma 4

We introduce independent binary variables r_j, $j = 1$ to M, so that $r_j = 1$ with probability $1-\gamma$ and $r_j = -1$ with probability γ. Define

$$Q_{i,\gamma}^{+} = \sum_{j=1}^{M} \log\left(1 + sgn(r_j y_j) sgn(u_{ij}) e^{-(K-1)w_{ij}}\right) = \sum_{j=1}^{M} \log\left(1 + sgn\left(r_j y_j / s_{ij}\right) e^{-(K-1)w_{ij}}\right)$$

Note that $sgn(r_j u_{ij}) = 1$ with probability $1/2(1-\gamma) + 1/2(\gamma) = 1/2$, hence it has the same distribution as $sgn(u_{ij})$. Following the proof of Lemma 2, we can derive

$$\begin{aligned}
&\mathbf{Pr}\left(Q_{i,\gamma}^{+} > \epsilon M/K, x_i = 0\right)\\
=&\mathbf{Pr}\left(\sum_{j=1}^{M} \log\left(1 + sgn(r_j y_j / s_{ij}) \exp\left(-(K-1)w_{ij}\right)\right) > \epsilon M/K, x_i = 0\right)\\
=&\mathbf{Pr}\left(\sum_{j=1}^{M} \log\left(1 + sgn(r_j S_j / s_{ij}) \exp\left(-(K-1)w_{ij}\right)\right) > \epsilon M/K\right)\\
=&\mathbf{Pr}\left(\sum_{j=1}^{M} \log\left(1 + sgn(r_j u_{ij}) \exp\left(-(K-1)w_{ij}\right)\right) > \epsilon M/K\right)\\
=&\mathbf{Pr}\left(\prod_{j=1}^{M} \left(1 + sgn(r_j u_{ij}) \exp\left(-(K-1)w_{ij}\right)\right) > e^{\epsilon M/K}\right)\\
=&\mathbf{Pr}\left(\prod_{j=1}^{M} \left(1 + sgn(u_{ij}) \exp\left(-(K-1)w_{ij}\right)\right) > e^{\epsilon M/K}\right)
\end{aligned}$$

At this point, it becomes the same as the problem in Lemma 2, hence we complete the proof.

D Proof of Lemma 5

$$\begin{aligned}
&\mathbf{Pr}\left(Q_{i,\gamma}^{+} < \epsilon M/K, x_i > 0\right)\\
=&\mathbf{Pr}\left(\prod_{j=1}^{M} \left(1 + sgn(r_j y_j / s_{ij}) \exp\left(-(K-1)w_{ij}\right)\right)^{-t} > \exp\left(-t\epsilon M/K\right), x_i > 0\right)\\
\leq&\exp\left(t\epsilon M/K\right) E^{M}\left(\left(1 + sgn(r_j y_j / s_{ij}) \exp\left(-(K-1)w_{ij}\right)\right)^{-t}; x_i > 0\right)
\end{aligned}$$

Consider $\alpha \to 0$. We study $sgn(r_j y_j/s_{ij}) = sgn\left(x_i r_j + r_j\theta_i S_j/s_{ij}\right)$, where $S_j, s_{ij} \sim S(\alpha,1)$ i.i.d. Let $T_{ij} = sgn(r_j y_j/s_{ij})\exp\left(-(K-1)w_{ij}\right)$. As $\alpha \to 0$

$$\begin{aligned}
T_{ij} =& sgn\left(x_i r_j + r_j\theta_i sgn(U_j) sgn(u_{ij})\left(\frac{w_{ij}}{W_j}\right)^{1/\alpha}\right)e^{-(K-1)w_{ij}}\\
=& sgn\left(x_i r_j + r_j sgn(U_j) sgn(u_{ij})\left((K-1)\frac{w_{ij}}{W_j}\right)^{1/\alpha}\right)e^{-(K-1)w_{ij}}\\
=& \begin{cases} sgn(r_j x_i)e^{-(K-1)w_{ij}} & \text{if } (K-1)w_{ij} < W_j \\ sgn(r_j u_{ij})e^{-(K-1)w_{ij}} & \text{if } (K-1)w_{ij} > W_j \end{cases}
\end{aligned}$$

Thus,

$$\begin{aligned}
& E\left(\left(1 + sgn(y_j/s_{ij})\exp\left(-(K-1)w_{ij}\right)\right)^{-t}; x_i > 0\right)\\
=&(1-\gamma)E\left\{\int_0^{W_j/(K-1)}\left(1+\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\} + \gamma E\left\{\int_0^{W_j/(K-1)}\left(1-\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\}\\
&+\frac{1}{2}E\left\{\int_{W_j/(K-1)}^{\infty}\left(1+\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\} + \frac{1}{2}E\left\{\int_{W_j/(K-1)}^{\infty}\left(1-\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\}\\
=&\frac{1}{2}\left\{\int_0^{\infty}\left(1+\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\} + \frac{1}{2}\left\{\int_0^{\infty}\left(1-\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\}\\
+&\left(\frac{1}{2}-\gamma\right)E\left\{\int_0^{W_j/(K-1)}\left(1+\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\}\\
&-\left(\frac{1}{2}-\gamma\right)E\left\{\int_0^{W_j/(K-1)}\left(1-\exp\left(-(K-1)u\right)\right)^{-t}e^{-u}du\right\}\\
=&\frac{1}{2}\int_0^1\left(1+u^b\right)^{-t}du + \frac{1}{2}\int_0^1\left(1-u^b\right)^{-t}e^{-u}du - \left(\frac{1}{2}-\gamma\right)\int_0^{\infty}e^{-w}\int_{w/b}^1\left[\left(1-u^b\right)^{-t} - \left(1+u^b\right)^{-t}\right]dudw
\end{aligned}$$

Again, for convenience, we denote $b = K-1$. As shown in the proof of Lemma 3, we have

$$\frac{1}{2}\int_0^1(1+u^b)^{-t}du + \frac{1}{2}\int_0^1(1-u^b)^{-t}du = 1 + \frac{t(t+1)}{(2b+1)2!} + \frac{t(t+1)(t+2)(t+3)}{(4b+1)4!} + ...$$

For the other term, we have

$$\begin{aligned}
&\int_0^{\infty}e^{-w}\int_{w/b}^1\left[\left(1-u^b\right)^{-t} - \left(1+u^b\right)^{-t}\right]dudw\\
=&2\int_0^{\infty}e^{-w}\int_{e^{-w/b}}^1\left[tu^b + t(t+1)(t+2)u^{3b}/3! + t(t+1)(t+2)(t+3)(t+4)u^{5b}/5! + ..\right]dudw\\
=&2\left[\frac{t}{b+1} + \frac{t(t+1)(t+2)}{(3b+1)3!} + \frac{t(t+1)(t+2)(t+3)(t+4)}{(5b+1)5!} + ...\right]\\
-&2\int_0^{\infty}e^{-w}\left[\frac{t}{b+1}(e^{-w/b})^{b+1} + \frac{t(t+1)(t+2)}{(3b+1)3!}(e^{-w/b})^{3b+1} + \frac{t(t+1)(t+2)(t+3)(t+4)}{(5b+1)5!}(e^{-w/b})^{5b+1} + ...\right]dw\\
=&2\left[\frac{t}{b+1} + \frac{t(t+1)(t+2)}{(3b+1)3!} + \frac{t(t+1)(t+2)(t+3)(t+4)}{(5b+1)5!} + ...\right]\\
-&2\left[\frac{t}{b+1}\frac{b}{2b+1} + \frac{t(t+1)(t+2)}{3!(3b+1)}\frac{b}{4b+1} + \frac{t(t+1)(t+2)(t+3)(t+4)}{5!(5b+1)}\frac{b}{6b+1} + ...\right]\\
=&2\left[\frac{t}{2b+1} + \frac{t(t+1)(t+2)}{3!(4b+1)} + \frac{t(t+1)(t+2)(t+3)(t+4)}{5!(6b+1)} + ...\right]
\end{aligned}$$

Combining the results yields

$$
\begin{aligned}
& E\left((1+sgn(y_j/s_{ij})\exp\left(-(K-1)w_{ij}\right))^{-t}; x_i > 0\right) \\
= & \left[1 + \frac{t(t+1)}{(2b+1)2!} + \frac{t(t+1)(t+2)(t+3)}{(4b+1)4!} + ...\right] \\
& - (1-2\gamma)\left[\frac{t}{2b+1} + \frac{t(t+1)(t+2)}{3!(4b+1)} + \frac{t(t+1)(t+2)(t+3)(t+4)}{5!(6b+1)} + ...\right]
\end{aligned}
$$

Therefore, we can write

$$
\mathbf{Pr}\left(Q^+_{i,\gamma} < \epsilon M/K, x_i > 0\right) \leq \exp\left(-\frac{M}{K}H_4(t;\epsilon,K,\gamma)\right)
$$

where

$$
H_4(t;\epsilon,K,\gamma) = -\epsilon t - K\log\left[1 + \sum_{n=2,4,6...}^{\infty}\frac{1}{n(K-1)+1}\prod_{l=0}^{n-1}\frac{t+l}{n-l} - \sum_{n=1,3,5...}^{\infty}\frac{1-2\gamma}{(n+1)(K-1)+1}\prod_{l=0}^{n-1}\frac{t+l}{n-l}\right]
$$

$$
H_4(t;\epsilon,\infty,\gamma) = -\epsilon t - \left[\sum_{n=2,4,6...}^{\infty}\frac{1}{n}\prod_{l=0}^{n-1}\frac{t+l}{n-l} - \sum_{n=1,3,5...}^{\infty}\frac{1-2\gamma}{(n+1)}\prod_{l=0}^{n-1}\frac{t+l}{n-l}\right]
$$