

Appendix

Proof of Theorem 1 *Proof:* The moment form of Lemma 1 can be represented as [18],

$$\begin{aligned} \mathbb{E}(h_1^{r_1} h_2^{r_2} \dots h_n^{r_n}) = \\ \frac{1}{\Gamma(r)} \int_0^\infty u^{r-1} e^{-\sum_{i=n+1}^k \Psi_i(u)} \prod_{j \in [n]} (-1)^{r_j} \frac{d^{r_j}}{du^{r_j}} e^{-\Psi_j(u)} du. \end{aligned} \quad (19)$$

We use the above general form of the moments to compute and diagonalize the following moment tensors,

$$\begin{aligned} \mathbf{M}_2^{(\mathbf{h})} &= \mathbb{E}(\mathbf{h} \otimes \mathbf{h}) + \eta \mathbb{E}(\mathbf{h}) \otimes \mathbb{E}(\mathbf{h}), \quad (20) \\ \mathbf{M}_3^{(\mathbf{h})} &= \mathbb{E}(\mathbf{h} \otimes \mathbf{h} \otimes \mathbf{h}) \\ &\quad + \eta_1 \mathbb{E}(\mathbf{h} \otimes \mathbf{h}) \otimes \mathbb{E}(\mathbf{h}) \\ &\quad + \eta_2 \mathbb{E}(\mathbf{h} \otimes \mathbb{E}(\mathbf{h}) \otimes \mathbf{h}) \\ &\quad + \eta_3 \mathbb{E}(\mathbf{h}) \otimes \mathbb{E}(\mathbf{h} \otimes \mathbf{h}) \\ &\quad + \eta_4 \mathbb{E}(\mathbf{h}) \otimes \mathbb{E}(\mathbf{h}) \otimes \mathbb{E}(\mathbf{h}). \quad (21) \end{aligned}$$

Setting the off-diagonal entries of Equations (20) and (21) to 0 and get the following set of equations

$$\begin{aligned} \mathbb{E}(h_i h_j) + \eta \mathbb{E}(h_i) \mathbb{E}(h_j) &= 0 \quad \text{for } i \neq j, \quad (22) \\ \mathbb{E}(h_i h_j h_l) \\ &\quad + \eta_1 \mathbb{E}(h_i h_j) \mathbb{E}(h_l) \\ &\quad + \eta_2 \mathbb{E}(h_i h_l) \mathbb{E}(h_j) \\ &\quad + \eta_3 \mathbb{E}(h_j h_l) \mathbb{E}(h_i) \\ &\quad + \eta_4 \mathbb{E}(h_i) \mathbb{E}(h_j) \mathbb{E}(h_l) = 0 \\ &\quad \text{for } i \neq j \neq l = 0, \quad (23) \end{aligned}$$

$$\begin{aligned} \mathbb{E}(h_i^2 h_l) \\ &\quad + \eta_1 \mathbb{E}(h_i^2) \mathbb{E}(h_l) \\ &\quad + \eta_2 \mathbb{E}(h_i h_l) \mathbb{E}(h_i) \\ &\quad + \eta_3 \mathbb{E}(h_i h_l) \mathbb{E}(h_i) \\ &\quad + \eta_4 \mathbb{E}(h_i) \mathbb{E}(h_i) \mathbb{E}(h_l) = 0 \\ &\quad \text{for } i \neq l. \quad (24) \end{aligned}$$

Writing the moments using Equation (19), assuming $\Phi_i(u) = \alpha_i \Psi(u)$, we get the following weights by some simple algebraic manipulations,

$$\eta = \frac{\int_0^\infty u e^{-\alpha_0 \Psi(u)} \left(\frac{d}{du} \Psi(u) \right)^2 du}{\left(\int_0^\infty e^{-\alpha_0 \Psi(u)} \frac{d}{du} \Psi(u) du \right)^2} \quad (25)$$

$$\eta_1 = \eta_2 = \eta_3$$

$$\begin{aligned} &\frac{1}{2} \int_0^\infty u^2 e^{-\alpha_0 \Psi(u)} \frac{d^2}{du^2} \Psi(u) \frac{d}{du} \Psi(u) du \\ &= - \frac{\int_0^\infty u e^{-\alpha_0 \Psi(u)} \frac{d^2}{du^2} \Psi(u) du \int_0^\infty e^{-\alpha_0 \Psi(u)} \frac{d}{du} \Psi(u) du}{\int_0^\infty u e^{-\alpha_0 \Psi(u)} \frac{d^2}{du^2} \Psi(u) du \int_0^\infty e^{-\alpha_0 \Psi(u)} \frac{d}{du} \Psi(u) du} \quad (26) \end{aligned}$$

$$\eta_4 = \frac{f(\psi(u))}{\left(\int_0^\infty e^{-\alpha_0 \Psi(u)} \frac{d}{du} \Psi(u) du \right)^3} \quad (27)$$

Where

$$\begin{aligned} f(\psi(u)) &= -\frac{1}{2} \int_0^\infty u^2 e^{-\alpha_0 \Psi(u)} \left(\frac{d}{du} \Psi(u) \right)^3 du \\ &\quad + (\eta_1 + \eta_2 + \eta_3) \int_0^\infty u e^{-\alpha_0 \Psi(u)} \left(\frac{d}{du} \Psi(u) \right)^2 du \\ &\quad \cdot \int_0^\infty e^{-\alpha_0 \Psi(u)} \frac{d}{du} \Psi(u) du \quad (28) \end{aligned}$$

Setting $v = \eta$, $v_1 = \eta_1 = \eta_2 = \eta_3$ and $v_2 = \eta_4$ and defining

$$\Omega(m, n, p) := \int_0^\infty u^m \frac{d^n}{du^n} \Psi(u) \left(\frac{d}{du} \Psi(u) \right)^p e^{-\alpha_0 \Psi(u)} du, \quad (29)$$

the set of weights v , v_1 and v_2 have the following form,

$$v = \frac{\Omega(1, 1, 1)}{\left(\Omega(0, 1, 0) \right)^2}, \quad (30)$$

$$v_1 = -\frac{\Omega(2, 2, 1)}{2\Omega(1, 2, 0)\Omega(0, 1, 0)}, \quad (31)$$

$$v_2 = \frac{-0.5\Omega(2, 1, 2) + 3v_1\Omega(1, 1, 1)\Omega(0, 1, 0)}{\left(\Omega(0, 1, 0) \right)^3}. \quad (32)$$

$$(33)$$

Weights v , v_1 and v_2 ensure that moment tensors $\mathbf{M}_2^{(\mathbf{h})}$ and $\mathbf{M}_3^{(\mathbf{h})}$ form diagonal tensors. Therefore they can be represented as,

$$\mathbf{M}_2^{(\mathbf{h})} = \sum_{i \in [k]} \kappa_i \mathbf{e}_i^{\otimes 2}, \quad (34)$$

$$\mathbf{M}_3^{(\mathbf{h})} = \sum_{i \in [k]} \lambda_i \mathbf{e}_i^{\otimes 3}, \quad (35)$$

where,

$$\kappa_i = \mathbb{E}[h_i^2] + v \mathbb{E}[h_i]^2, \quad (36)$$

$$\lambda_i = \mathbb{E}[h_i^3] + 3v_1 (\mathbb{E}[h_i^2] \mathbb{E}[h_i]) + v_2 (\mathbb{E}[h_i]^3). \quad (37)$$

The exchangeability assumption on the word space gives,

$$\mathbb{E}[\mathbf{x}_1] = \mathbb{E}(\mathbb{E}[\mathbf{x}_1 | \mathbf{h}]) = \mathbf{A} \mathbb{E}(\mathbf{h}), \quad (38)$$

$$\mathbb{E}[\mathbf{x}_1 \otimes \mathbf{x}_2] = \mathbb{E}(\mathbb{E}[\mathbf{x}_1 \otimes \mathbf{x}_2 | \mathbf{h}]) = \mathbf{A} \mathbb{E}(\mathbf{h} \otimes \mathbf{h}) \mathbf{A}^\top, \quad (39)$$

$$\begin{aligned}\mathbb{E}[\mathbf{x}_1 \otimes \mathbf{x}_2 \otimes \mathbf{x}_3] &= \mathbb{E}(\mathbb{E}[\mathbf{x}_1 \otimes \mathbf{x}_2 \otimes \mathbf{x}_3 | \mathbf{h}]) \\ &= \mathbb{E}[\mathbf{h} \otimes \mathbf{h} \otimes \mathbf{h} | \mathbf{A}, \mathbf{A}, \mathbf{A}].\end{aligned}\quad (40)$$

Therefore,

$$\mathbf{M}_2 = \mathbf{A} \mathbf{M}_2^{(\mathbf{h})} \mathbf{A}^\top = \sum_{j \in [k]} \kappa_j (\mathbf{a}_j \otimes \mathbf{a}_j), \quad (41)$$

$$\mathbf{M}_3 = \mathbf{M}_3^{(\mathbf{h})}(\mathbf{A}, \mathbf{A}, \mathbf{A}) = \sum_{j \in [k]} \lambda_j (\mathbf{a}_j \otimes \mathbf{a}_j \otimes \mathbf{a}_j) \quad (42)$$

□

Extended results

Table 4: NID Top 10 Words for NYtimes, K = 20

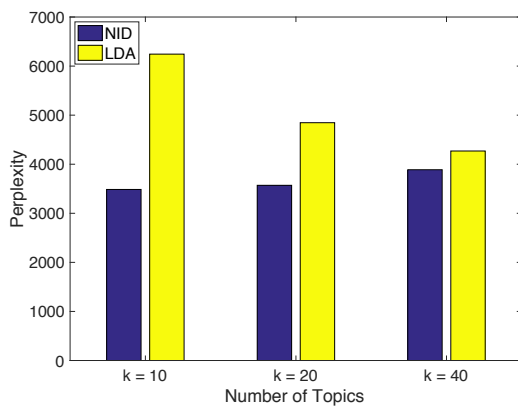
Topic	Top Words in descending order of importance
1	seeded, soldier, firestone, bobby-braswell, michigan-state, actresses, gary-william, preview, school-district, netanyahu
2	diane, question, newspaper, copy, fall, held, tonight, send, guard, slugged
3	abides, acclimate, acetate, alderman, analogues, annexing, ansar, antitax, antitobacco, argyle
4	percent, school, quarter, company, taliban, high, stock, race, companies, john-mccain
5	test, deal, contract, tiger-wood, question, houston-chronicle, copy, won, seattle-post-intelligencer ,tax
6	tonight, diane, question, newspaper, file, copy, fall, slugged, onlytest, xxx
7	company, com, market, stock, won, los-angeles-daily-new, business, eastern, web, commentary
8	abides, acclimate, acetate, alderman, analogues, annexing, ansar, antitax, antitobacco, argyle
9	company, game, run, los-angeles-daily-new, percent, team, season, stock, companies, games
10	working-girl, abides, acclimate, acetate, alderman, analogues, annexing, ansar, antitax, antitobacco
11	diane, newspaper, fall, tonight, question, held, copy, bush, slugged, police
12	hurricanes, policies, surgery, productivity, courageous, emergency, singapore, orange-bowl, regarding, telecast
13	abides, acclimate, acetate, alderman, analogues, annexing, ansar, antitax, antitobacco, argyle
14	company, com, won, stock, market, eastern, commentary, business, web, deal
15	company, stock, market, business, investor, technology, analyst, cash, sell, executives
16	tonight, question, diane, file, newspaper, copy, fall, slugged, onlytest, xxx
17	defense, held, children, fight, assistant, surgery, michael-bloomberg, worker, bird, omar
18	percent, company, stock, companies, quarter, school, market, analyst, high, corp
19	school, student, yard, released, guard, premature, teacher, touchdown, publication, leader
20	school, percent, student, yard, high, taliban, flight, air, afghanistan, plan

Table 5: NID top 10 Words for Pubmed, K = 10

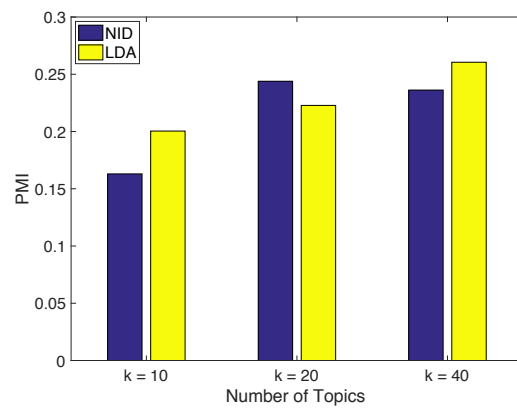
Topic	Top Words in descending order of importance
1	protein, region, dna, family, sequence, gene, form-12, analysis.abstract, model, tumoural
2	cell, mice.abstract, expression.abstract, activity.abstract, primary, tumor, antigen, human, t-cell, vitro
3	tumor, treatment, receptor, lesional, children-a, effect.abstract, factor, rat1, renal-cell, response-1
4	patient, treatment, therapy, clinical, disease, level.abstract, effect.abstract, treated, tumor, surgery
5	activity.abstract, rat1, concentration, dna, human, effect.abstract, exposure.abstract, animal-based, reactional, inhibition.abstract
6	patient, children-a, women.abstract, treatment, level.abstract, syndrome, disordered, disease, year-1, therapy
7	effect.abstract, receptor, level.abstract, rat1, mutational, gene, concentration, women.abstract, insulin, expression.abstract
8	acid, strain, concentration, women.abstract, test, pregnancy-a, drug, system-a, function.abstract, water
9	strain, protein, system-a, muscle, mutational, species, growth, diagnosis-based, analysis.abstract, gene
10	infection.abstract, hospital, programed, strain, medical, alpha, information, health, children-a, data.abstract

Table 6: Spectral LDA top 10 Words for NYtimes, K = 20

Topic	Top Words in descending order of importance
1	newspaper, question, copy, fall, diane, chante-lagon, kill, mandatory, drug, patient
2	held, guard, send, publication, released, advisory, premature, attn-editor, undatelined, washington-datelined
3	los-angeles-daily-new, slugged, com, xxx, www, x-x-x, web, information, site, eastern
4	million, shares, offering, boston-globe, debt, public, initial, player, bill, contract
5	onlytest, point, tax, case, court, lawyer, police, minutes, death, shot
6	held, released, publication, guard, advisory, premature, send, attn-editor, undatelined, washington-datelined
7	com, information, www, web, eastern, daily, commentary, business, separate, marked
8	boston-globe, spot, file, killed, tonight, women, earlier, article, george-bush, incorrectly
9	million, shares, offering, debt, public, initial, player, contract, bond, revenue
10	boston-globe, spot, file, held, killed, attn-editor, earlier, article, court, women
11	percent, market, stock, point, quarter, economy, rate, women, growth, companies
12	boston-globe, spot, file, tonight, killed, earlier, article, women, incorrectly, news-feature
13	held, guard, publication, released, send, advisory, premature, attn-editor, undatelined, washington-datelined
14	los-angeles-daily-new, slugged, xxx, new-york, x-x-x, fund, bush, goal, king, evening
15	tonight, copy, question, diane, fall, newspaper, russia, terrorist, russian, black
16	slugged, los-angeles-daily-new, xxx, new-york, x-x-x, bush, run, school, inning, student
17	onlytest, file, film, onlyendpar, movie, new-york, seattle-pi, los-angeles, sport, patient
18	los-angeles-daily-new, slugged, xxx, x-x-x, student, inning, send, program, enron, game
19	los-angeles-daily-new, slugged, xxx, new-york, x-x-x, fund, evening, program, student, enron
20	test, houston-chronicle, hearst-news-service, seattle-post-intelligencer, ignore, patient, kansas-city, yard, race, doctor



(a) Perplexity score



(b) Pointwise Mutual Information (PMI)

Figure 5: Perplexity and PMI scores for the NYtimes dataset across different number of topics