

## Supplementary Material: Rate Optimal Estimation for High Dimensional Spatial Covariance Matrices

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### Detailed Results of Numerical Study.

In this part, we provide more detailed results of the numerical study in Section 4. More setting of the parameters are simulated than what was presented in Section 4. The following table ?? summarizes the relative errors of the estimators. And the result shows the improvement of the double tapering(and banding) estimator against the sample covariance matrix in most cases, particularly in the high-dimensional ( $p$ ) cases.

p	n	1.1		1.2		1.3		1.4		1.5	
		band	taper	band	taper	band	taper	band	taper	band	taper
400	250	0.67	0.67	0.61	0.67	0.61	0.69	0.59	0.72	0.63	0.75
	500	0.82	0.73	0.72	0.7	0.66	0.71	0.63	0.73	0.65	0.75
	1000	1.06	0.85	0.9	0.76	0.76	0.74	0.69	0.74	0.68	0.76
	2000	1.42	1.04	1.18	0.87	0.93	0.79	0.81	0.77	0.73	0.77
	3000	1.71	1.2	1.41	0.96	1.06	0.84	0.92	0.79	0.79	0.78
900	250	0.52	0.52	0.46	0.51	0.45	0.52	0.43	0.54	0.46	0.56
	500	0.66	0.59	0.56	0.55	0.5	0.54	0.46	0.55	0.47	0.56
	1000	0.87	0.72	0.72	0.62	0.6	0.57	0.53	0.57	0.5	0.57
	2000	1.18	0.92	0.96	0.75	0.75	0.64	0.64	0.6	0.56	0.59
	3000	1.42	1.09	1.16	0.85	0.87	0.71	0.73	0.64	0.61	0.61
1600	250	0.43	0.42	0.37	0.41	0.36	0.41	0.34	0.42	0.36	0.44
	500	0.55	0.5	0.46	0.45	0.41	0.43	0.37	0.43	0.37	0.45
	1000	0.73	0.62	0.6	0.52	0.49	0.47	0.43	0.45	0.4	0.46
	2000	1	0.81	0.8	0.64	0.62	0.54	0.52	0.49	0.45	0.48
	3000	1.21	0.96	0.97	0.74	0.73	0.6	0.6	0.53	0.5	0.49
2500	250	0.36	0.36	0.31	0.34	0.3	0.34	0.28	0.35	0.29	0.36
	500	0.47	0.43	0.39	0.38	0.34	0.36	0.31	0.36	0.31	0.37
	1000	0.63	0.54	0.51	0.45	0.41	0.4	0.36	0.38	0.33	0.38
	2000	0.86	0.71	0.69	0.56	0.53	0.46	0.44	0.42	0.38	0.4
	3000	1.04	0.85	0.83	0.65	0.62	0.52	0.51	0.45	0.42	0.42
3600	250	0.31	0.31	0.27	0.29	0.25	0.29	0.24	0.3	0.25	0.31
	500	0.41	0.38	0.34	0.33	0.29	0.31	0.26	0.31	0.26	0.32
	1000	0.55	0.48	0.44	0.39	0.36	0.35	0.31	0.33	0.28	0.32
	2000	0.76	0.64	0.6	0.5	0.46	0.41	0.38	0.36	0.33	0.34
	3000	0.92	0.77	0.73	0.58	0.54	0.46	0.44	0.39	0.36	0.36

Table 1: A more comprehensive simulation study under similar settings in section 4. Multivariate Gaussian data is generated according to the covariance matrix  $\Sigma$  as defined in (15). The dimension  $p$  ranges from 400 to 3600, and the decay rate  $a$  ranges from 1.1 to 1.5. The multiplicative constant  $c$  is set to be 10. All the relative error results are based on the average of 100 replications. Smaller (less than one) values correspond to better performance.