SUPPLEMENTARY MATERIAL

A. Spline Filters Construction

We provide in this section a step-by-step construction of the proposed spline filters. First, in Fig. 4 we show the Hermite cubic spline that will be used as building blocks our filters. As can be seen, it is a cubic polynomial defined on a closed interval. Its parameters are uniquely defined by specifying the values of the polynomial at the boundaries as well as the values of the derivative of the polynomial at the boundaries. Then, in Fig. 5 we demonstrate how one leverages multiple



Figure 4. Hermite cubic spline: cubic polynomial on a close interval

Hermite cubic splines to construct the spline filters. The first step is to concatenate the Hermite cubic splines on a uniform partition of a closed interval. Each region leverages a Hermite cubic spline and we denote as spline filter the piecewise Hermite cubic spline function. In order to enforce the spline filter to be in the space of the considered filter (here wavelets),



Figure 5. Concatenation of Hermite cubic splines

one has to impose continuity and smoothness by constraining the values that the Hermite cubic splines of each region,

Fig. 6. In fact, by specifying that neighboring Hermite cubic splines have the same values at the shared boundary we reach smoothness. In addition, we require a localized and centered spline filter. This is imposed by constraining the values of the



Figure 6. Insuring continuity and smoothness of the filter

Hermite cubic splines as demonstrated in Fig. 7. With the derived mother filter, it is now possible to sample the filter-bank



Figure 7. From the smooth filter to wavelet

that can be used in place of standard filter of a convolutional layer of a deep network. To do so, the analytical expression is simply evaluated over a uniform sampling grid. Each grid will sample a filter and the filter-bank is sampled with different grids, each with different number of points. The more points in the grid the more dilated will be the filters.



Figure 8. From the mother wavelet to the filter-bank