Appendix: Dependent Relational Gamma Process Models for Longitudinal Networks

A. Dependent Relational Gamma Process

Let $G = \sum_{k=1}^{\infty} r_k \delta_{\boldsymbol{\theta}_k}$ be a draw from a gamma process $\mathrm{GaP}(G_0, c)$, where c is a scale parameter and G_0 is a finite and continuous base measure over a complete separable metric space Θ (Ferguson, 1973). A model based on the gamma process has an inherent shrinkage mechanism since the number of atoms with weights greater than $\epsilon > 0$ follows $\mathrm{Poisson}(\gamma_0 \int_{\epsilon}^{\infty} \mathrm{d}r c r^{-1} e^{-cr})$, where $\gamma_0 \equiv G_0(\Theta)$ is the total mass under the base measure.

Given G drawn from a gamma process, we exploit the relational gamma process proposed by (Zhou, 2015),

$$\Lambda \mid G \sim \mathrm{RGaP}(G, \xi, \beta),$$

a sample from which is expressed as

$$\Lambda \mid G = \sum_{k=1}^{\infty} \sum_{k'=1}^{\infty} \lambda_{kk'} \delta_{(\boldsymbol{\theta}_k, \boldsymbol{\theta}_{k'})},$$

where $\xi > 0$ and $\beta > 0$,

$$\lambda_{kk'} \sim \begin{cases} \mathrm{Gamma}(\xi r_k, \beta), & \text{if } k = k' \\ \mathrm{Gamma}(r_k r_{k'}, \beta), & \text{otherwise} \end{cases}$$

Given Λ drawn from a relational gamma process, we generate a set of independent binary variables $\{b_k^{(t)}\}_{k=1}^{\infty}$. The new process

$$\omega_{lk} \sim \mathcal{NIG}(0, 1, 1), \;\; \phi_k \sim \mathrm{Cat}(\phi_1^*, \ldots, \phi_D^*),$$
$$b_k^{(t)} \sim \mathrm{Bernoulli}\left(\sigma\left\{\omega_{0k} + \sum_{l=1}^{T} \omega_{lk} \exp[-\phi_k (t-l)^2]\right\}\right),$$
$$\Lambda^{(t)} = \sum_{k=1}^{\infty} \sum_{k'=1}^{\infty} b_k^{(t)} b_{k'}^{(t)} \lambda_{kk'} \delta_{(\boldsymbol{\theta}_k, \boldsymbol{\theta}_{k'})},$$

is well-defined by the mapping theorem for the Poisson processes (Kingman, 1993); see (Foti et al., 2013) for a complete proof.

B. MCMC Inference

Notation. When expressing the full conditionals for Gibbs sampling, we use the shorthand "–" to denote all other variables. We use "·" as an index summation shorthand, e.g., $x_{\cdot j} = \sum_i x_{ij}$.

The following definition and results are exploited to derive our closed-form Gibbs sampling update equations.

Negative-Binomial Distribution. A negative-binomial (NegBin) distributed random variable $y \sim \mathrm{NegBin}(r, p)$ can be generated from a gamma mixed Poisson distribution as, $y \sim \mathrm{Poisson}(\lambda)$ and $\lambda \sim \mathrm{Gamma}(r, \frac{p}{1-p})$ by marginalizing over λ.

Poisson-Logarithmic Bivariate Distribution. The Poisson-logarithmic bivariate distributed variable (y, l) with $y \sim \mathrm{NegBin}(y; r, p)$ and a Chinese restaurant table (CRT) distributed variable $l \sim \mathrm{CRT}(l; y, r)$ can equivalently be expressed as a sum-logarithmic (SumLog) variable and a Poisson variable, i.e., $y = \sum_{s=1}^{l} u_s$ with $u_s \sim \mathrm{Logarithmic}(p)$, and $l \sim \mathrm{Poisson}(-r \ln(1-p))$ (Zhou et al., 2015).

Sampling latent count $m_{ij}^{(t)}$: We sample the latent count $m_{ij}^{(t)}$ as

$$(m_{ij}^{(t)} \mid -) \sim A_{ij}^{(t)} \mathrm{Poisson}_{+}\left(\sum_{k=1}^{K} \sum_{k'=1}^{K} z_{ik}^{(t)} \lambda_{kk'}^{(t)} z_{jk'}^{(t)}\right). \tag{6}$$

We note that we can re-express the latent Poisson count $m_{ij}^{(t)}$ as

$$m_{ij}^{(t)} = \sum_{k=1}^{K} \sum_{k'=1}^{K} m_{ikk'j}^{(t)},$$

where $m_{ikk'j}^{(t)} \sim \mathrm{Poisson}(z_{ik}^{(t)} \lambda_{kk'}^{(t)} z_{jk'}^{(t)})$. We define $m_{ik\cdot\cdot}^{(t)} \equiv \sum_{j \neq i} \sum_{k'} m_{ikk'j}^{(t)}$. Via the Poisson additive property, we have $m_{ik\cdot\cdot}^{(t)} \sim \mathrm{Poisson}(z_{ik}^{(t)} \psi_{ik}^{(t)})$, where $\psi_{ik}^{(t)} \equiv \sum_{j \neq i} \sum_{k'} z_{jk'}^{(t)} \lambda_{kk'}^{(t)}$.

Sampling latent subcount $m_{ikk'j}^{(t)}$: To update the node-group memberships $\{z_{ik}^{(t)}\}_{i,k,t}$ and group-group interaction weights $\{\lambda_{kk'}^{(t)}\}_{k,k',t}$, we need to partition the count $m_{ij}^{(t)}$ into the sub counts $\{m_{ikk'j}^{(t)}\}_{k,k'}$, where $m_{ikk'j}^{(t)}$ measures the interaction strength between nodes i and j due to their associations to groups k and k', respectively. Via the Poisson-multinomial equivalence, we sample the latent subcounts $m_{ikk'j}^{(t)}$ as

$$(\{m_{ikk'j}^{(t)}\} \mid -) \sim \mathrm{Multinomial}\left(m_{ij}^{(t)}; \frac{\{\lambda_{kk'}^{(t)} z_{ik}^{(t)} z_{jk'}^{(t)}\}}{\sum_{k=1}^{K} \sum_{k'=1}^{K} \lambda_{kk'}^{(t)} z_{ik}^{(t)} z_{jk'}^{(t)}} \right). \tag{7}$$

Sampling thinning variable $b_k^{(t)}$: If $\sum_i m_{ik\cdot\cdot}^{(t)} > 0$, we set $b_k^{(t)} = 1$, and if $\sum_i m_{ik\cdot\cdot}^{(t)} = 0$, we sample $b_k^{(t)}$ by the following process: we define fictitious latent counts $\varpi_k^{(t)} \sim \mathrm{Poisson}(r_k \xi \rho_{kk})$ disregarding $b_k^{(t)}$ to determine whether $\sum_i m_{ik\cdot\cdot}^{(t)} = 0$ because group k has been thinned or because group k has not been observed at time t. Hence, we sample $b_k^{(t)}$ when $\sum_i m_{ik\cdot\cdot}^{(t)} = 0$ as

1. If $\varpi_k^{(t)} = 0$, we sample $b_k^{(t)}$ as

$$\begin{aligned} p(b_k^{(t)} = 1 \mid \varpi_{kk}^{(t)} = 0) &\propto p(b_k^{(t)} = 1)\mathrm{Poisson}(0; r_k \xi \rho_{kk}), \\ p(b_k^{(t)} = 0 \mid \varpi_{kk}^{(t)} = 0) &\propto p(b_k^{(t)} = 0)\mathrm{Poisson}(0; r_k \xi \rho_{kk}). \end{aligned} \tag{8}$$

2. If $\varpi_k^{(t)} > 0$, we sample $b_k^{(t)}$ as

$$p(b_k^{(t)} = 1 \mid \varpi_{kk}^{(t)} > 0) \propto p(b_k^{(t)} = 1) \left[1 - \mathrm{Poisson}(0; r_k \xi \rho_{kk})\right]. \tag{9}$$

Sampling kernel weights $\boldsymbol{\omega}$: The normal-inverse-gamma prior placed over ω_{lk} can be equivalently generated from the following process by introducing auxiliary variables $\{\vartheta_{lk}\}$:

$$\begin{aligned} \vartheta_{lk} &\sim \mathrm{Gamma}(1, 1), \\ \omega_k &\sim \mathcal{N}(0, \Sigma_\vartheta), \end{aligned}$$

where $\omega_k = (\omega_{1k}, \ldots, \omega_{Lk})$ and $\Sigma_\vartheta = \mathrm{diag}(\vartheta_{0k}, \ldots, \vartheta_{Lk})$.
Let $\mathcal{K}_{tk} = (1, \mathcal{K}(t, t_1, \phi_k), \ldots, \mathcal{K}(t, t_L, \phi_k))^{\mathrm{T}}$ be the vector of the kernels evaluated at time t. We sample $\{\omega_{lk}\}$ exploiting a Pólya-gamma data augmentation technique (Polson et al., 2013) for logistic regression by introducing auxiliary variables as

$$(\tilde{b}_{kt} \mid -) \sim \mathrm{PG}(1, \mathcal{K}_{tk}^{\mathrm{T}} \omega_k),$$

where $\mathrm{PG}(a, b)$ denotes the Pólya-gamma distribution with $b \in \mathbb{R}$ and $a > 0$. Let $\Omega(\tilde{b}_k)$ denote the $T \times T$ diagonal matrix whose t-th diagonal element is $\tilde{b}_{kt}$, and let $\mu_k = (b_k^{(1)} - 1/2, \ldots, b_k^{(T)} - 1/2)^{\mathrm{T}}$. The conditional distribution of ω_k is

$$(\omega_k \mid -) \sim \mathcal{N}(\mu_{\omega_k}, \Sigma_{\omega_k}), \tag{10}$$

where $\Sigma_{\omega_k} = (\Sigma_\vartheta^{-1} + \mathcal{K}_{tk}^{\mathrm{T}} \Omega(\tilde{b}_k) \mathcal{K}_{tk})^{-1}$ and $\mu_{\omega_k} = \Sigma_{\omega_k} \mathcal{K}_{tk}^{\mathrm{T}} \mu_k$.

We sample ϑ_{lk} from its conditional posterior via the gamma normal conjugacy as

$$(\vartheta_{lk} \mid -) \sim \text{Gamma}\Big(\frac{3}{2}, 1 + \frac{1}{2}\omega_{lk}^2\Big).$$

Sampling kernel width ϕ: We uniformly draw ϕ_k from a fixed dictionary $\{\phi_1^*, \dots, \phi_D^*\}$ of size D, and hence sample ϕ_k as

$$p(\phi_k = \phi_d^* \mid -) \propto \frac{1}{D} \prod_{t \in \mathcal{T}} \big(P_{\phi_d^*}(t)\big)^{b_k^{(t)}} \, \big(1 - P_{\phi_d^*}(t)\big)^{1-b_k^{(t)}} \tag{11}$$

where the thinning function is denoted as a function of ϕ_d^* since the values of all the other variables are fixed as

$$P_{\phi_d^*}(t) = \sigma\Big\{\omega_{0k} + \sum_{l=1}^{T} \omega_{lk} \exp[-\phi_k (t-l)^2]\Big\}.$$

Augmenting and marginalizing the gamma Markov processes: We start from $t = T$ because none of the latent memberships $\{z_{ik}^{(t)}\}_{t=1}^{T-1}$ at previous times depend on $z_{ik}^{(T)}$ in their prior specifications,

$$m_{ik\cdot\cdot}^{(T)} \sim \text{Poisson}(z_{ik}^{(T)} \psi_{ik}^{(T)}), \qquad z_{ik}^{(T)} \sim \text{Gamma}(z_{ik}^{(T-1)}, \tau).$$

Marginalizing over $z_{ik}^{(T)}$ via the gamma Poisson conjugacy, we obtain

$$m_{ik\cdot\cdot}^{(T)} \sim \text{NegBin}(z_{ik}^{(T-1)}, \eta_{ik}^{(T)}),$$

where $\eta_{ik}^{(T)} \equiv \frac{\psi_{ik}^{(T)}}{\tau + \psi_{ik}^{(T)}}$.

To marginalize over $z_{ik}^{(T-1)}$, we introduce an auxiliary variable $\hat{m}_{ik}^{(T)} \sim \text{CRT}(m_{ik\cdot\cdot}^{(T)}, z_{ik}^{(T-1)})$. Then, we augment $m_{ik\cdot\cdot}^{(T)}$ under its compound Poisson representation as

$$m_{ik\cdot\cdot}^{(T)} = \sum_{l=1}^{\hat{m}_{ik}^{(T)}} u_l, \quad u_l \sim \text{Logarithmic}(\eta_{ik}^{(T)}), \quad \hat{m}_{ik}^{(T)} \sim \text{Poisson}\Big[-z_{ik}^{(T-1)} \ln(1 - \eta_{ik}^{(T)})\Big].$$

Since we already have $m_{ik\cdot\cdot}^{(T-1)} \sim \text{Poisson}(z_{ik}^{(T-1)} \psi_{ik}^{(T-1)})$, we immediately obtain

$$\begin{aligned} \tilde{m}_{ik}^{(T-1)} &\equiv \hat{m}_{ik}^{(T)} + m_{ik\cdot\cdot}^{(T-1)} \\ &\sim \text{Poisson}\Big(z_{ik}^{(T-1)} \Big[\psi_{ik}^{(T-1)} - \ln(1 - \eta_{ik}^{(T)})\Big]\Big). \end{aligned} \tag{12}$$

Combining the Poisson likelihood in Eq. (12) with the gamma prior placed on $z_{ik}^{(T-1)}$, we can marginalize over $z_{ik}^{(T-1)}$ and have

$$\tilde{m}_{ik}^{(T-1)} \sim \text{NegBin}\Big(z_{ik}^{(T-2)}, \eta_{ik}^{(T-1)}\Big),$$

where $\eta_{ik}^{(T-1)} \equiv \frac{\psi_{ik}^{(T-1)} - \ln(1-\eta_{ik}^{(T)})}{\tau + \psi_{ik}^{(T-1)} - \ln(1-\eta_{ik}^{(T)})}$.

We then recursively introduce $\hat{m}_{ik}^{(T-1)} \sim \text{CRT}(\tilde{m}_{ik}^{(T-1)}, z_{ik}^{(T-2)})$, and augment $\tilde{m}_{ik}^{(T-1)}$ under its compound Poisson representation as

$$\tilde{m}_{ik}^{(T-1)} = \sum_{l=1}^{\hat{m}_{ik}^{(T-1)}} u_l, \quad u_l \sim \text{Logarithmic}(\eta_{ik}^{(T-1)}), \quad \hat{m}_{ik}^{(T-1)} \sim \text{Poisson}\Big[-z_{ik}^{(T-2)} \ln(1 - \eta_{ik}^{(T-1)})\Big].$$

Marginalizing over $z_{ik}^{(T-2)}$ as we did for $z_{ik}^{(T-1)}$ yields

$$\tilde{m}_{ik}^{(T-2)} \sim \text{NegBin}\Big(z_{ik}^{(T-3)}, \eta_{ik}^{(T-2)}\Big).$$

Repeatedly exploiting the same procedure from $t = T$ to 1, we augment each latent membership $z_{ik}^{(t)}$ with an auxiliary variable $\hat{m}_{ik}^{(t)}$, which backwardly propagates the summarized information from time t to $t-1$. Via the augmented parameter space, we can straightforwardly obtain closed-form conditional posteriors for $\{z_{ik}^{(t)}\}_{t=1}^{T}$ using the gamma Poisson conjugacy.

Sampling node-group memberships $Z^{(1:T)}$: We can sample the auxiliary variables $\hat{m}_{ik}^{(t)}$ and update $\eta_{ik}^{(t)}$ backwardly from $t = T$ to 1 as

$$\hat{m}_{ik}^{(t)} \sim \text{CRT}(\hat{m}_{ik}^{(t+1)} + m_{ik\cdot\cdot}^{(t)}, z_{ik}^{(t-1)}), \tag{13}$$

$$\eta_{ik}^{(t)} = \frac{\psi_{ik}^{(t)} - \ln(1 - \eta_{ik}^{(t+1)})}{\tau + \psi_{ik}^{(t)} - \ln(1 - \eta_{ik}^{(t+1)})}, \tag{14}$$

where we have $z_{ik}^{(0)} \equiv \theta_{ik}$, $\hat{m}_{ik}^{(T+1)} = 0$ and $\eta_{ik}^{(T+1)} = 0$.
We then sample θ_{ik} and $z_{ik}^{(t)}$ forwardly from $t = 1$ to T as

$$(\theta_{ik} \mid -) \sim \text{Gamma}\Big[1 + \hat{m}_{ik}^{(1)}, 1 - \ln(1 - \eta_{ik}^{(1)})\Big], \tag{15}$$

$$(z_{ik}^{(t)} \mid -) \sim \text{Gamma}\Big[\hat{m}_{ik}^{(t+1)} + z_{ik}^{(t-1)} + m_{ik\cdot\cdot}^{(t)}, \tau + \psi_{ik}^{(t)} - \ln(1 - \eta_{ik}^{(t+1)})\Big], \qquad t \in \mathcal{T}. \tag{16}$$

Marginalizing over Λ, r: We define the latent Poisson count

$$m_{\cdot kk'\cdot}^{(\cdot)} \equiv 2^{-\delta_{kk'}} \sum_t \sum_i \sum_{j \neq i} m_{ikk'j}^{(t)},$$

where $\delta_{kk'} = 1$ if $k = k'$, and $\delta_{kk'} = 0$ otherwise. Via the Poisson additive property, we have

$$m_{\cdot kk'\cdot}^{(\cdot)} \sim \text{Poisson}(\lambda_{kk'}\rho_{kk'}),$$

where $\rho_{kk'} \equiv \sum_t b_k^{(t)} b_{k'}^{(t)} \sum_i \sum_{j\neq i} z_{ik}^{(t)} z_{jk'}^{(t)}$.
As we have the prior specification $\lambda_{kk'} \sim \text{Gamma}(r_k \xi^{\delta_{kk'}} r_{k'}^{1-\delta_{kk'}}, \beta)$, marginalizing over $\lambda_{kk'}$ yields

$$m_{\cdot kk'\cdot}^{(\cdot)} \sim \text{NegBin}(r_k \xi^{\delta_{kk'}} r_{k'}^{1-\delta_{kk'}}, \chi_{kk'}),$$

where $\chi_{kk'} \equiv \frac{\rho_{kk'}}{\beta + \rho_{kk'}}$.

To marginalize over r_k, we introduce an auxiliary variable:

$$l_{kk'} \sim \text{CRT}(m_{\cdot kk'\cdot}^{(\cdot)}, r_k \xi^{\delta_{kk'}} r_{k'}^{1-\delta_{kk'}}), \tag{17}$$

and then re-express the joint distribution over $m_{\cdot kk'\cdot}^{(\cdot)}$ and $l_{kk'}$ as

$$m_{\cdot kk'\cdot}^{(\cdot)} = \sum_{l=1}^{l_{kk'}} u_l, \quad u_l \sim \text{Logarithmic}(\chi_{kk'}), \quad l_{kk'} \sim \text{Poisson}[-r_k \xi^{\delta_{kk'}} r_{k'}^{1-\delta_{kk'}} \ln(1 - \chi_{kk'})].$$

Via the Poisson additive property, we have

$$l_{k\cdot} \equiv \sum_{k'} l_{kk'} \sim \text{Poisson}[-r_k \sum_{k'} \xi^{\delta_{kk'}} r_{k'}^{1-\delta_{kk'}} \ln(1 - \chi_{kk'})].$$

Sampling group interaction weights Λ: Via the gamma Poisson conjugacy, we sample $\lambda_{kk'}$ from its conditional posterior as

$$(\lambda_{kk'} \mid -) \sim \text{Gamma}\Big[m_{\cdot kk'\cdot}^{(\cdot)} + r_k \xi^{\delta_{kk'}} r_{k'}^{1-\delta_{kk'}}, \beta + \rho_{kk'}\Big]. \tag{18}$$

Algorithm 1 Gibbs sampling algorithm for DRGPM

Input: relational data $\{A^{(t)}\}_{t=1}^{T}$, iterations $\mathcal{J}$.
Initialize the maximum number of groups K, hyperparameters γ_0, β, c, τ.
for $iter = 1$ to $\mathcal{J}$ **do**
  Sample $\{m_{ij}^{(t)}\}_{i,j,t}$ for non-zero edges (Eq. 6)
  Sample $\{m_{ikk'j}^{(t)}\}_{i,j,k,k',t}$ (Eq. 7) and update
    $m_{\cdot kk'\cdot}^{(\cdot)} = \sum_t \sum_{i,j\neq i} m_{ikk'j}^{(t)}$
    $m_{ik\cdot\cdot}^{(t)} = \sum_{j\neq i,k'} m_{ikk'j}^{(t)}$
  for $t = 1$ to T **do**
    Sample $\{b_k^{(t)}\}_k$ (Eqs. 8; 9)
  end for
  Sample $\{\omega_k\}_k$ (Eq. 10) and ϕ (Eq. 11)
  for $t = T$ to 1 **do**
    Sample $\{\hat{m}_{ik}^{(t)}\}_{i,k}$ (Eq. 13) and update $\{\eta_{ik}^{(t)}\}_{i,k}$ (Eq. 14)
  end for
  Sample $\{\theta_{ik}\}_{i,k}$ (Eq. 15)
  for $t = 1$ to T **do**
    Sample $\{z_{ik}^{(t)}\}_{i,k}$ (Eq. 16)
  end for
  Sample $\{l_{kk'}\}_{k,k'}$ (Eq. 17) and update
    $\rho_{kk'} = \sum_{t,i,j\neq i} b_k^{(t)} b_{k'}^{(t)} z_{ik}^{(t)} z_{jk'}^{(t)}, \quad \chi_{kk'} = \frac{\rho_{kk'}}{\rho_{kk'}+\beta}$
  Sample $\{\lambda_{kk'}\}_{k,k'}$ (Eq. 18), $\{r_k\}_k$ (Eq. 19), and ξ (Eq. 20)
end for
Output posterior means: $\{z_{ik}^{(1:T)}\}_{i,k}, \{\theta_{ik}\}_{i,k}, \{r_k\}_k, \xi, \{\lambda_{kk'}\}_{k,k'}, \{b_k^{(1:T)}\}_k$.

where $\rho_{kk'} \equiv \sum_t b_k^{(t)} b_{k'}^{(t)} \sum_i \sum_{j\neq i} z_{ik}^{(t)} z_{jk'}^{(t)}$.

Sampling group weight r_k: Using the gamma-Poisson conjugacy, we sample r_k as

$$(r_k \mid -) \sim \text{Gamma}\Big[\frac{\gamma_0}{K} + \sum_{k'} l_{kk'}, c - \sum_{k'} \xi^{\delta_{kk'}} r_{k'}^{1-\delta_{kk'}} \ln(1-\chi_{kk'})\Big]. \tag{19}$$

Sampling ξ: Using the gamma-Poisson conjugacy, we sample

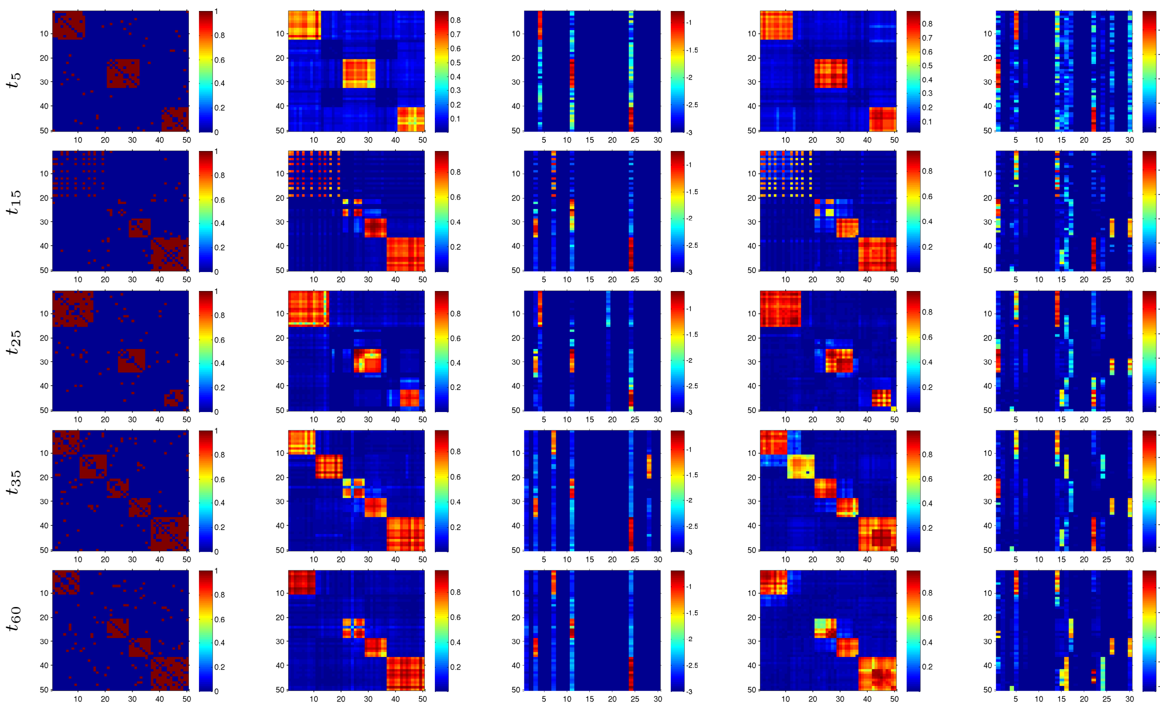
$$(\xi \mid -) \sim \text{Gamma}\Big[1 + \sum_k l_{kk}, 1 - \sum_k r_k \ln(1-\chi_{kk})\Big]. \tag{20}$$

The full procedure of our Gibbs sampling algorithm is summarized in Algorithm 1.

C. Additional Experimental Results

C.1. Simulation Study

We also compare DRGPM based on the thinned CRM framework against DPGM that only models time-evolving node memberships to clarify the contributions by modelling group birth/death dynamics. To this end, we apply DPGM on the same synthetic data used in Section 5.1. The inferred link probabilities and node-group associations $z_{ik}^{(t)}\lambda_{kk}$ by DPGM are depicted in columns (d) and (e) of Figure 6, respectively. The inferred link probabilities and node-group associations by DRGPM are depicted in columns (b) and (c) of Figure 6, respectively. We note that both DPGM and DRGPM infer fewer numbers of groups than D-GPPF because dynamic node-group connections are explicitly modelled by time-evolving node memberships in the former two methods. In particular, we notice that DPGM unavoidably generates some redundant groups that lack interpretability. This is due to that DPGM assumes the inferred group weights to be static throughout the whole time period.



(a) (b) (c) (d) (e)

Figure 6. Five selected snapshots of the simulated network as shown in column (a). The link probabilities inferred by DRGPM and DPGM are shown in columns (b) and (d), respectively. The association weights of each node (row variable) to the groups (column variable), as shown in columns (c) and (e), can be calculated as $z_{ik}^{(t)}\lambda_{kk}^{(t)}$ for DRGPM and $z_{ik}^{(t)}\lambda_{kk}$ for DPGM, respectively. The pixel values are displayed on $\log_{10}$ scale.

C.2. Additional Results for The MID Dataset

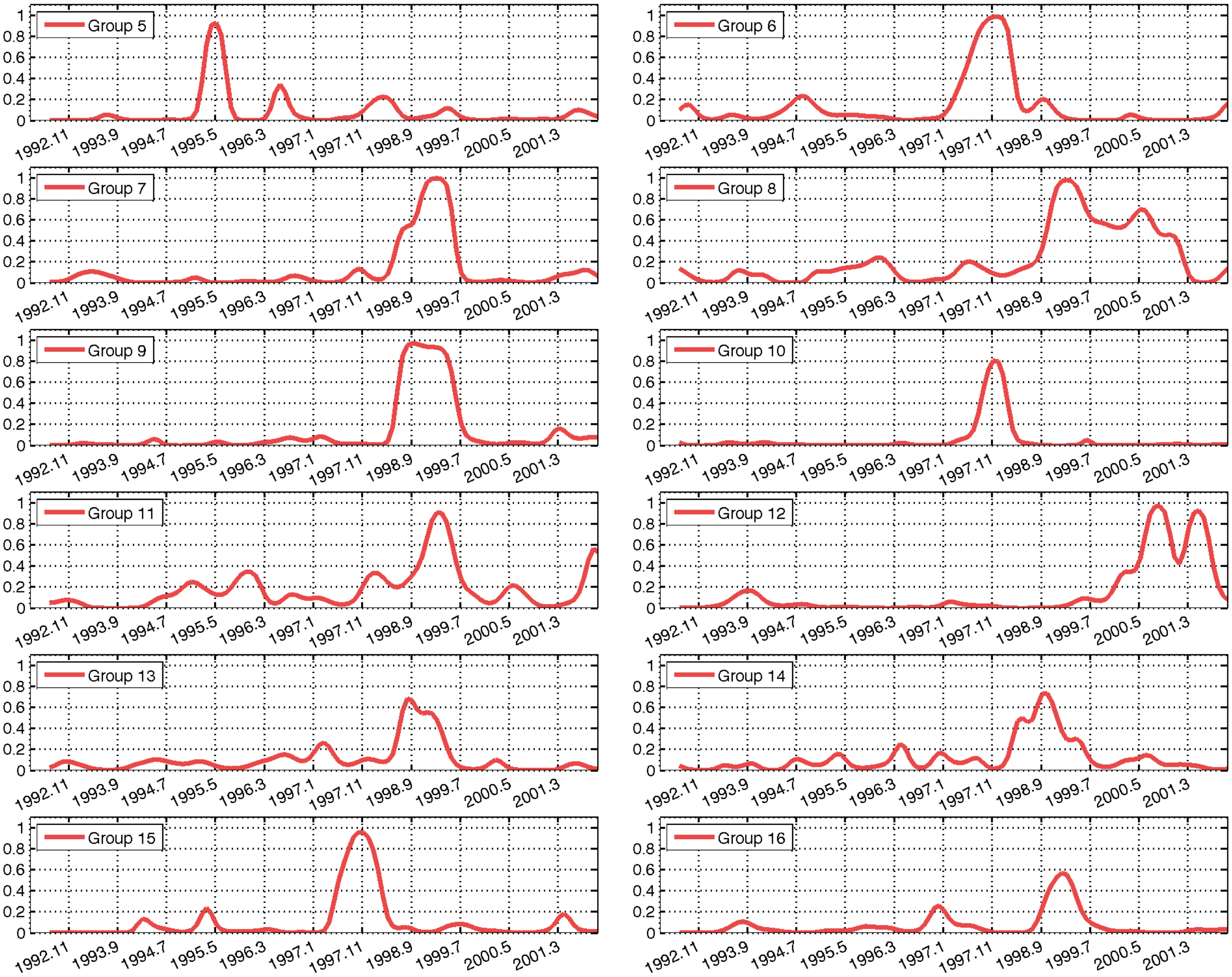


Figure 7. The activity (mean of $b_k^{(t)}$) of the selected groups inferred by DRGPM from the MID network.

Table 5. The top 20 nodes associated to each of the selected groups as shown in Fig. 7 from the MID network. The highest node memberships to the corresponding selected groups throughout the whole period are reported for each node in parentheses.

Group	Country
5	Taiwan (0.47), China (0.20), Thailand (0.13), Philippines (0.10), Cambodia (0.05), Vietnam (0.03), Turkey (0.01) Togo (0.01)
6	Sierra Leone (0.90), Nigeria (0.04), Guinea (0.04), Ghana (0.02)
7	Norway (0.42), Canada (0.28), Portugal (0.14), Turkey (0.14), United States of America (0.02)
8	Uganda (0.57), Rwanda (0.41), Eritrea (0.01), Congo (0.01), Bahrain (0.01)
9	Yugoslavia (0.88), United States of America (0.03), Denmark (0.02), Russia (0.02), Canada (0.02), Haiti (0.01) Bangladesh (0.01), Cuba (0.01)
10	Iraq (0.45), North Korea (0.22), Russia (0.17), Cyprus (0.10), Greece (0.06)
11	United States of America (0.34), Turkey (0.25), United Kingdom (0.21), South Korea (0.12), Denmark (0.02) Trinidad and Tobago (0.02), Japan (0.02), Norway (0.01), Honduras (0.01)
12	Israel (0.98) , El Salvador (0.01) , United States of America (0.01)
13	Portugal (0.31), Turkey (0.17), United Kingdom (0.12), Denmark (0.11), Belgium (0.10), Norway (0.10), Albania (0.08)
14	South Korea (0.30), United States of America (0.22), Vietnam (0.18), Afghanistan (0.11), Norway (0.07), Mongolia (0.05), Denmark (0.04), Peru (0.03)
15	Turkey (0.47), United States of America (0.29), South Korea (0.09), Iran (0.08), Georgia (0.06)
16	Albania (0.25), Portugal (0.23), Canada (0.21), Denmark (0.18), Norway (0.13)