A DERIVATION FOR THE PARAMETER INFERENCE ALGORITHM

Here we lay out the details of the full conditional distributions used in step 1 and step 2 in the sampling algorithm in Section 3.2.

Since not all the players are on the court all the time in a basketball game, parameters relevant to a player are inferred using only the observations at the moments when he was playing.

A.1 Full Conditionals for the Covariate Coefficients (Step 1)

For each player pair \((i, j)\), let \(\theta_{i,j}\) be the vector of all log-risks player \(i\) passing to \(j\) across all the games in the data. For each entry \(\theta_{i,j,g}(t)\) in the vector (suppose that the corresponding game is \(g\)), let

\[
\tilde{\theta}_{i,j,g}(t) = \theta_{i,j,g}(t) - u_i^T v_{j,g},
\]

and we have

\[
\tilde{\theta}_{i,j,g}(t) = X_{i,j,g}(t)^T \beta_{i,j} + \epsilon_{i,j,g}(t). 
\]

Re-writing the above in matrix form (by stacking together all the entries across all the games), we get

\[
\tilde{\theta}_{i,j} = X_{i,j}^T \beta_{i,j} + \epsilon_{i,j}. 
\]

Then under the Gaussian model for independent errors \(\epsilon_{i,j,g}(t)\) and a normal prior for \(\beta_{i,j}\),

\[
\epsilon_{i,j,g}(t) \sim N(0, 1), \quad \beta_{i,j} \sim N(b_{i,j}, \tau^2 I_7) \tag{4}
\]

\(p(\beta_{i,j}|\Theta, U, V)\) is a multivariate normal distribution,

\[
\beta_{i,j}|\Theta, U, V \sim N(m_{i,j}, M_{i,j}) \tag{5}
\]

where

\[
M_{i,j} = (I_7/\tau^2 + X_{i,j}^T X_{i,j})^{-1}, \\
M_{i,j} = M_{i,j}(b_{i,j}/\tau^2 + X_{i,j}^T \tilde{\theta}_{i,j}). \tag{6}
\]

In Eq \(\ref{4}\), we adopt a diffuse prior for \(\beta_{i,j}\), with \(\tau = 1000\) and \(b_{i,j}\) as the OLS estimate for \(\beta_{i,j}\) assuming \(U = V = 0\).

A.2 Full Conditionals for the Multiplicative Latent Effects (Step 2)

For each game \(g\) and each player \(i\) who played in game \(g\), let \(TM_g(i)\) be the set of players who shared the court with \(i\) in game \(g\). For \(j \in TM_g(i)\) and time \(t\) at which \(i\) possessed the ball and \(j\) was on the court, we have

\[
\tilde{\theta}_{i,j,g}(t) = u_i^T v_{j,g} + \epsilon_{i,j,g}(t), \tag{7}
\]

where

\[
\tilde{\theta}_{i,j,g}(t) = \theta_{i,j,g}(t) - X_{i,j,g}(t)^T \beta_{i,j}. \tag{8}
\]

Re-writing Eq \(\ref{7}\) in matrix form, we have

\[
\tilde{\theta}_{i,g} = V_{TM_g(i),g}^T u_{i,g} + \epsilon_{i,g}, \tag{9}
\]

where \(V_{TM_g(i),g}\) is the matrix with each row being a receiver-specific latent effect vector \(v_{j,g}\) in Eq \(\ref{7}\).

Under a normal prior \(u_{i,g} \sim N(0, I_R)\), \(p(U_g[i]|\Theta, V, \beta)\) is a multivariate normal distribution,

\[
U_g[i]|\Theta, V, \beta \sim N(w_{i,g}, W_{i,g}) \tag{10}
\]

where

\[
W_{i,g} = (I_R + V_{TM_g(i),g}^T V_{TM_g(i),g})^{-1}, \\
w_{i,g} = W_{i,g}(IR + V_{TM_g(i),g}^T \tilde{\theta}_{i,g}). \tag{11}
\]

Very similarly, for each game \(g\) and each player \(j\) who played in game \(g\), let \(PO_g(j)\) be the set of players who shared the court with \(j\) and ever possessed the ball in game \(g\). For \(i \in PO_g(j)\) and time \(t\) at which \(j\) was the ballcarrier and \(i\) was on the court, we have (with a slight abuse of notation)

\[
\tilde{\theta}_{i,j,g}(t) = u_i^T v_{j,g} + \epsilon_{i,j,g}(t). \tag{12}
\]
Re-writing Eq (12) in matrix form, we have

\[ \bar{\theta}_{j,g} = U^T PO_{g}(j,g) v_{j,g} + \epsilon_{j,g}, \] (13)

where \( U PO_{g}(j,g) \) is the matrix with each row being a sender-specific latent effect vector \( u_{i,g} \) in Eq (12).

Under a normal prior \( v_{j,g} \sim N(0, I_R) \), \( p(V_g \mid \Theta, U, \beta) \) is a multivariate normal distribution,

\[ V_g \mid \Theta, U, \beta \sim N(w'_{j,g}, W'_{j,g}), \] (14)

where

\[ W'_{j,g} = (I_R + U^T PO_{g}(j,g) U PO_{g}(j,g))^{-1}, \]
\[ w'_{j,g} = W'_{j,g}(I_R + U^T PO_{g}(j,g) \bar{\theta}_{j,g}). \] (15)

B ADDITIONAL PLOTS OF MULTIPLICATIVE LATENT EFFECTS

B.1 Additional Plots for the AME Model in Section 2

Figure 1: Learned multiplicative sender-specific effects (in blue) and receiver-specific effects (in red) by the AME model in a lost game (top) and a won game (bottom). Each latent effect vector corresponds to a two-dimensional coordinate represented by a player’s id code.

Figure 2: Products of sender- and receiver-specific effects learned in the AME model for all player pairs in a loss (top) and in a win (bottom). Darker color indicates higher frequency of passes. The level of interaction between teammates is significantly higher in a win than in a loss. Furthermore, there are significant passing behavior anomalies in the lost game: player 121034 strongly favors 109412 and 126160 as ball receivers but ignores 109415 as a potential receiver, which is not the case in the victory where his passing choices are more balanced.
B.2 Additional Plots for the Real Data
Experiments in Section 4

Figure 3: Passing decision multiplicative latent factors in a loss (top) versus in a victory (bottom). Sender-specific effects are marked with “S” plus player id codes in blue, and receiver-specific effects are marked with “R” plus player id codes in red.

(a) Number of passes between players in a lost game.

(b) Number of passes between players in a successful game.

Figure 4: Raw passing counts between home team players in a loss versus in a victory. Darker color indicates more passes made from a sender to a receiver. It is obvious that these two plots are disparate from the plots in Fig.5 in the main text, and that the differences between a win and a loss are much harder to observe if only the raw passing counts are examined.