Supplementary Materials

We supplement the proof of *Lemma* 4.1 in Section 4 of the main paper.

Lemma 4.1. *If we can arrange* d *outputs into a* Q*-mode tensor with an equal dimension in each mode, and* $Qd^{\frac{1}{Q}} \leq N$*, the time complexity of HOGPR is* $\mathcal{O}(N^2 d)$ *— linear to the number of outputs. Such* Q *always exists when* $d \leq e^{\frac{N}{e}}$*.*

Proof. First, since $d_1 = \ldots = d_Q$ and $d = \prod_{k=1}^{Q} d_k$, we have each $d_k = d^{\frac{1}{Q}}$. Then we have $\sum_{k=1}^{Q} d_k = Qd^{\frac{1}{Q}} \leq N$ and $Nd(\sum_{k=1}^{Q} d_k + N) \leq 2N^2 d$. Therefore, the time complexity is $\mathcal{O}(N^2 d)$. Second, if we allow continuous Q, and take the gradient of the function, $f(Q) = Qd^{\frac{1}{Q}}$, we have

$$\frac{\nabla f(Q)}{\nabla Q} = d^{\frac{1}{Q}}(1 - \frac{\log(d)}{Q}).$$

When $Q \leq \log(d)$, the gradient will be non-positive, and the function is monotonically decreasing. The function reaches the minimum at $Q = \log(d)$, and the minimum value is $e \cdot \log d$. Hence as long as $N \geq e \cdot \log d$ — equivalently, $d \leq e^{\frac{N}{e}}$, there must exists Q such that $f(Q) \leq N$, namely, $Qd^{\frac{1}{Q}} \leq N$. $\square$