Active Sequential Learning with Tactile Feedback: Supplementary material

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Supplementary terms

The terms $\mathbf{q}$, $\mathbf{Q}$, and $\mathbf{Z}$ introduced in Section 3.2.1 can be expanded as follows:

$$\begin{aligned}
(\mathbf{q}_m)_i &= \alpha_m^2 |\mathbf{\Sigma}_m(\mathbf{H}_m^{\boldsymbol{\theta}})^{-1}+\mathbf{I}|^{\frac{1}{2}} \\
&\times \exp(-\frac{1}{2}(\boldsymbol{\mu}-\boldsymbol{\tau}^i)^T(\mathbf{\Sigma}_m+\mathbf{H}_m^{\boldsymbol{\theta}})^{-1}(\boldsymbol{\mu}-\boldsymbol{\tau}^i)) \\
&\times \exp(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\chi}^i)^T(\mathbf{H}_m^{\mathbf{x}})^{-1}(\mathbf{x}-\boldsymbol{\chi}^i)) \\
(\mathbf{Q}_{mn})_{ij} &= \alpha_m^2\alpha_n^2|((\mathbf{H}_m^{\boldsymbol{\theta}})^{-1}+(\mathbf{H}_n^{\boldsymbol{\theta}})^{-1})\mathbf{\Sigma}+\mathbf{I}|^{\frac{1}{2}} \\
&\times \exp(-\frac{1}{2}(\mathbf{z}^{mj}-\mathbf{z}^{nj})^T\mathbf{U}_{mn}^{-1}(\mathbf{z}^{mi}-\mathbf{z}^{nj})) \\
&\times \exp(-\frac{1}{2}(\boldsymbol{\tau}^i-\boldsymbol{\tau}^j)^T(\mathbf{H}_m^{\boldsymbol{\theta}}+\mathbf{H}_n^{\boldsymbol{\theta}})^{-1}(\boldsymbol{\tau}^i-\boldsymbol{\tau}^j)) \\
&\times \exp(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\chi}^i)^T(\mathbf{H}_m^{\mathbf{x}})^{-1}(\mathbf{x}-\boldsymbol{\chi}^i)) \\
&\times \exp(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\chi}^j)^T(\mathbf{H}_n^{\mathbf{x}})^{-1}(\mathbf{x}-\boldsymbol{\chi}^j)) \\
(\mathbf{Z}_m)_{\bullet i} &= (\mathbf{q}_m)_i(\mathbf{\Sigma}^{-1}+(\mathbf{H}_m^{\boldsymbol{\theta}})^{-1})^{-1} \\
&\times (\mathbf{\Sigma}^{-1}\boldsymbol{\mu}+(\mathbf{H}_m^{\boldsymbol{\theta}})^{-1}\boldsymbol{\tau}^i)
\end{aligned}$$

where $\mathbf{z}^{mi} = \mathbf{H}_m^{\boldsymbol{\theta}}(\boldsymbol{\tau}^i-\boldsymbol{\mu})$, $\mathbf{R}_{mn} = (\mathbf{H}_m^{\boldsymbol{\theta}}+\mathbf{H}_n^{\boldsymbol{\theta}})^{-1}+\mathbf{\Sigma}$ and $\mathbf{U}_{mn} = (\mathbf{H}_m^{\boldsymbol{\theta}}+\mathbf{H}_n^{\boldsymbol{\theta}})^{-1}\mathbf{R}^{-1}\mathbf{\Sigma}(\mathbf{H}_m^{\boldsymbol{\theta}}+\mathbf{H}_n^{\boldsymbol{\theta}})^{-1}$. $(\mathbf{Z}_m)_{\bullet i}$ denotes the i-th column vector of matrix $\mathbf{Z}_m$.

The gradients are defined as:

$$\begin{aligned}
\frac{\partial(\mathbf{q}_m)_i(\mathbf{x})}{\partial\mathbf{x}} &= (\mathbf{q}_m)_i(\mathbf{H}_m^{\mathbf{x}})^{-1}(\mathbf{x}-\boldsymbol{\chi}^i) \\
\frac{\partial(\mathbf{Q}_{mn})_{ij}}{\partial\mathbf{x}} &= (\mathbf{Q}_{mn})_{ij}\big(-(\mathbf{H}_m^{\mathbf{x}})^{-1}(\mathbf{x}-\boldsymbol{\chi}^i) \\
&\quad -(\mathbf{H}_m^{\mathbf{x}})^{-1}(\mathbf{x}-\boldsymbol{\chi}^j)\big) \\
\frac{\partial(\mathbf{Z}_m)_{ij}}{\partial\mathbf{x}} &= \frac{\partial(\mathbf{q}_m)_i(\mathbf{x})}{\partial\mathbf{x}}(\mathbf{\Sigma}^{-1}+(\mathbf{H}_m^{\boldsymbol{\theta}})^{-1})^{-1} \\
&\quad \times (\mathbf{\Sigma}^{-1}\boldsymbol{\mu}+(\mathbf{H}_m^{\boldsymbol{\theta}})^{-1}\boldsymbol{\tau})
\end{aligned}$$