Supplementary Material for Replica Conditional Sequential Monte Carlo

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1. Validity of Replica cSMC

It is easy to see that the proposed update leaves $\bar{\pi}$ invariant. Let $M_{x_{1:T}^{(-k)}}(x_{1:T}^{(k)'}|x_{1:T}^{(k)})$ be the cSMC transition kernel used to update replica $x_{1:T}^{(k)}$, $k = 1, \ldots, K$, where $x_{1:T}^{(-k)} := (x_{1:T}^{(1)'}, \ldots, x_{1:T}^{(k-1)'}, x_{1:T}^{(k+1)}, \ldots, x_{1:T}^{(K)})$. The replica update is a composition of the $M_{x_{1:T}^{(-k)}}$ so we can write the replica cSMC transition kernel M as a product, $M(x_{1:T}^{(1:K)'}|x_{1:T}^{(1:K)}) = \prod_{k=1}^{K} M_{x_{1:T}^{(-k)}}(x_{1:T}^{(k)'}|x_{1:T}^{(k)})$.

The replica cSMC transition kernel M then leaves $\bar{\pi}$ invariant since we have

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\begin{aligned}
&\int \bar{\pi}(x_{1:T}^{(1:K)}) M(x_{1:T}^{(1:K)'}|x_{1:T}^{(1:K)}) dx_{1:T}^{(1:K)} \\
&= \int \prod_{k=1}^{K} p(x_{1:T}^{(k)}|y_{1:T}) M_{x_{1:T}^{(-k)}}(x_{1:T}^{(k)'}|x_{1:T}^{(k)}) dx_{1:T}^{(1:K)} \\
&= \int \left[\int p(x_{1:T}^{(1)}|y_{1:T}) M_{x_{1:T}^{(-1)}}(x_{1:T}^{(1)'}|x_{1:T}^{(1)}) dx_{1:T}^{(1)} \right] \\
&\times \prod_{k=2}^{K} p(x_{1:T}^{(k)}|y_{1:T}) M_{x_{1:T}^{(-k)}}(x_{1:T}^{(k)'}|x_{1:T}^{(k)}) dx_{1:T}^{(2:K)} \\
&= p(x_{1:T}^{(1)'}|y_{1:T}) \int \left[\int p(x_{1:T}^{(2)}|y_{1:T}) M_{x_{1:T}^{(-2)}}(x_{1:T}^{(2)'}|x_{1:T}^{(2)}) dx_{1:T}^{(2)} \right] \\
&\times \prod_{k=3}^{K} p(x_{1:T}^{(k)}|y_{1:T}) M_{x_{1:T}^{(-k)}}(x_{1:T}^{(k)'}|x_{1:T}^{(k)}) dx_{1:T}^{(3:K)} \\
&= p(x_{1:T}^{(1)'}|y_{1:T}) p(x_{1:T}^{(2)'}|y_{1:T}) \\
&\times \int \prod_{k=3}^{K} p(x_{1:T}^{(k)}|y_{1:T}) M_{x_{1:T}^{(-k)}}(x_{1:T}^{(k)'}|x_{1:T}^{(k)}) dx_{1:T}^{(3:K)} \\
&= \prod_{k=1}^{K} p(x_{1:T}^{(k)'}|y_{1:T}) \quad \text{(by induction)} \\
&= \bar{\pi}(x_{1:T}^{(1:K)'}).
\end{aligned}
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