Stochastic Model for Sunk Cost Bias - Supplementary material

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A MISSING PROOFS FROM SECTION 4

Claim A.1 *The function $\prod_{i=1}^{n-1}(1-p_i)\cdot(\sum_{i=1}^{n-1}p_i)$ attains its maximal value for $0 \le p_i \le 1$, when for every i, $p_i = 1/n$.*

Proof: We simply take partial derivatives and compare them to 0. To this end, it will be more convenient to use $q_i = 1 - p_i$ and take partial derivatives of the function

$$f(q_1,\dots,q_n) = \prod_{i=1}^{n-1} q_i \cdot (n - 1 - \sum_{i=1}^{n-1} q_i).$$

Observe that:

$$\frac{\partial f}{\partial q_i} = \prod_{j\neq i} q_j(n-1-\sum_{j\neq i} q_j) - 2\cdot\prod_j q_j$$

By comparing it to 0 and some rearranging we get that:

$$\begin{aligned} 2\cdot\prod_j q_j &= \prod_{j\neq i} q_j(n-1-\sum_{j\neq i} q_j) \\ \implies 2q_i &= n-1-\sum_{j\neq i} q_j \\ \implies q_i &= n-1-\sum_j q_j \end{aligned}$$

Thus,we have that for every i, q_i has the same value of $q_i = n - 1 - \sum_j q_j$ and to compute the value of q_i we can solve: $q = n - 1 - (n-1)q$ which implies that $q = \frac{n-1}{n}$. Thus, we have that in our original maximization problem, for every i, $p_i = 1/n$. □

Lemma A.2 *For any $n \ge 3$ the value of λ in Theorem 4.4 is smaller than 1.*

Proof: Recall that $\lambda = \frac{\pi_o}{(1-p)^{n-1}(n-1)c}$ where $\pi_o = \frac{1-(1-p)^{n-1}}{p}(p-c)$, $p = \frac{1}{n}$ and $c = \frac{1}{n} - \frac{1}{n^2}$. By plugging in the values of p, c and π_o we get that

$$\lambda = \frac{(1-(1-\frac{1}{n})^{n-1})}{n(1-\frac{1}{n})^{n+1}}$$

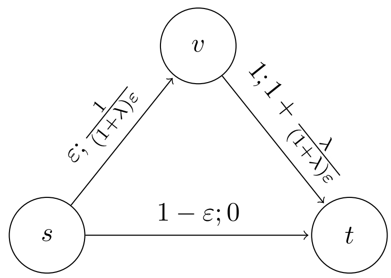

Figure 1: On each edge edge the left expression is the probability of taking the edge and the right number is the cost if the edge is taken. For $R = 1$, we have that $\pi_s = 0$ and $\pi_o = \frac{\lambda}{1+\lambda} \cdot R - \varepsilon$.

To show that $\lambda \leq 1$ it suffices to show that:

$$(1 - \frac{1}{n})^{n-1} + n(1 - \frac{1}{n})^{n+1} \geq 1$$

Let $f(n) = (1 - \frac{1}{n})^{n-1} + n(1 - \frac{1}{n})^{n+1}$. Observe that $f(3) = 28/27 > 1$. Thus, showing that $f(n)$ is increasing will complete the proof. Note that

$$f'(n) = \frac{\left(\frac{n-1}{n}\right)^n \left(\left(n^2 - n + 1\right) \ln\left(\frac{n-1}{n}\right) + 2n - 1\right)}{n-1}$$

and by using calculus one can show that it is indeed the case that $f'(n) > 0$ for any $n > 2$ which completes the proof. □

B THREE NODE INSTANCES

Claim B.1 *In an alternative model in which costs are positioned on the the edges. For any ε there exists a 3-node graph in which $\pi_s = 0$ and $\pi_o = \frac{\lambda}{1+\lambda} \cdot R - \varepsilon$.*

Proof: Consider the 3-node graph depicted in Figure 1. In this graph, it is clear that the optimal agent will not continue from v to t but the sophisticated agent will. Thus, the expected payoff of the optimal agent is:

$$\pi_o = (1 - \varepsilon) \cdot 1 - \varepsilon \cdot \frac{1}{(1 + \lambda)\varepsilon} = \frac{\lambda}{1 + \lambda} - \varepsilon$$

If a λ biased sophisticated agent will choose to traverse the graph he will always reach the target. Thus, its expected payoff will be:

$$1 - \varepsilon(\frac{1}{(1 + \lambda)\varepsilon} + \frac{\lambda}{(1 + \lambda)\varepsilon}) = 0$$

Thus, the payoff of the sophisticated agent is 0.

□

Claim B.2 *For any 3-node graph and any $\lambda \geq 0$, $\pi_s \geq \pi_o - \frac{2+\lambda-2\sqrt{1+\lambda}}{\lambda} \cdot R$ and this is tight.*

Proof: Consider the graph in figure 2. First, one can observe that the only two possible scenarios in which the optimal and the sophisticated agent will have different payoffs are:

- The optimal agent traverses the graph for a single step and the sophisticated agent continues.
- The optimal agent traverses the graph for a single step and the sophisticated agent is unwilling to start.

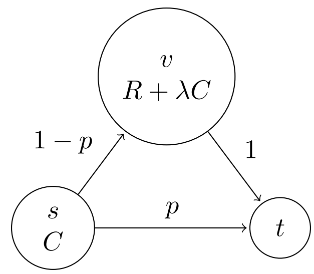

Figure 2: 3-node graph illustration for Claim B.2.

These are the only scenarios we should consider as if the optimal agent does not traverse the graph or continues at v then the sophisticated agent will do the same.

We begin by considering the scenario in which the optimal agent traverses the graph for a single step and the sophisticated agent continues. Observe that $\pi_o = pR - C$. Notice that if the sophisticated agent would start traversing the graph it would continue at v. Thus, its expected payoff for traversing the graph is $pR - C - (1-p)\lambda C \leq 0$. By rearranging we get that $p \leq \frac{C(1+\lambda)}{R+\lambda C}$. Thus, to maximize the expected payoff of the optimal agent, we set $p = \frac{C(1+\lambda)}{R+\lambda C}$ and get that:

$$\pi_o = \frac{C(1+\lambda)}{R+\lambda C} \cdot R - C$$

To maximize π_o we take a derivative with respect to C and compare it to 0:

$$\frac{\partial \pi_o}{\partial C} = \frac{R(1+\lambda)(R+\lambda C) - \lambda C R(1+\lambda)}{(R+\lambda C)^2} - 1 = \frac{R^2(1+\lambda)}{(R+\lambda C)^2} - 1$$

$$\frac{R^2(1+\lambda)}{(R+\lambda C)^2} - 1 = 0 \implies R^2(1+\lambda) = (R+\lambda C)^2$$

$$C = \frac{R}{\lambda}\left(\sqrt{1+\lambda} - 1\right)$$

which gives

$$\begin{aligned} p = \frac{R\left(\sqrt{1+\lambda}-1\right)(1+\lambda)}{\lambda(R + R(\sqrt{1+\lambda}-1))} &= \frac{\left(\sqrt{1+\lambda}-1\right)\sqrt{1+\lambda}}{\lambda} \\ &= \frac{1+\lambda-\sqrt{1+\lambda}}{\lambda} \end{aligned}$$

Therefore:

$$\begin{aligned} \pi_o &= \frac{1+\lambda-\sqrt{1+\lambda}}{\lambda} \cdot R - \frac{R}{\lambda}(\sqrt{1+\lambda}-1) \\ &= \frac{(2+\lambda-2\sqrt{1+\lambda})}{\lambda} \cdot R \end{aligned}$$

For $0 < \lambda \leq 1$ we get that $0 < \pi_o - \pi_s \leq 0.172R$.

Finally, we consider the scenario in which the sophisticated agent traverse the graph and continues at v while the optimal agent stops traversing the graph at v. We show that optimizing the payoff difference for this scenario get us to the same optimization problem as we just solved. Denote by $c(v)$ the cost at v. Since the sophisticated agent continues we have that $R - c(v) \geq -\lambda C \implies c(v) \leq R + \lambda C$. Also, since the expected payoff of the sophisticated agent is positive we have that:

$$\begin{aligned} \pi_s = pR - C + (1-p)(R - c(v)) > 0 &\implies \\ R + pc(v) - c(v) - C > 0 &\implies \\ c(v) &< \frac{R-C}{1-p} \end{aligned}$$

Consider the difference between the payoffs of the agents:

$$\pi_o - \pi_s = -(1-p)(R - c(v)) = (1-p)(c(v) - R)$$

Clearly, the difference is maximized for the maximal value of $c(v)$. Since $c(v) \leq \min\{\frac{R-C}{1-p}, R + \lambda C\}$ we get that this value is maximized when $\frac{R-C}{1-p} = R + \lambda C$ by rearranging we get that in this case $p = \frac{C(1+\lambda)}{R+\lambda C}$. Since in this case we have that:

$$\pi_o - \pi_s \leq (1-p)(\min\{\frac{R-C}{1-p}, R + \lambda C\} - R) = p \cdot R - C$$

This implies the exact optimization problem as in the first case, which completes the proof. □