Analysis of Langevin Monte Carlo from Poincaré to Log-Sobolev

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Abstract

Whereas prior works on sampling have largely focused on strongly log-concave targets, classically the continuous-time Langevin diffusion converges exponentially fast to its stationary distribution $\pi \propto \exp(-V)$ in chi-squared divergence under the sole assumption that π satisfies a Poincaré inequality. Using this fact to provide guarantees for the discrete-time Langevin Monte Carlo (LMC) algorithm, however, is considerably more challenging in the non-log-concave case due to the need for working with chi-squared or Rényi divergences.

In this work, we provide the first Rényi divergence convergence guarantees for LMC which allow for weak smoothness and do not require convexity or dissipativity conditions. Our complexity bounds for LMC to reach ε accuracy in Rényi divergence are summarized as follows:

(i) $\widetilde{O}(d/\varepsilon)$ iterations when π satisfies a log-Sobolev inequality and V is smooth;

(ii) $\widetilde{O}(d^2/\varepsilon)$ iterations when π is log-concave and V is smooth;

(iii) $\widetilde{O}(d^{(2/\alpha)\,(1+1/s)-1/s}/\varepsilon^{1/s})$ iterations when π satisfies either a Latała–Oleszkiewicz or a modified log-Sobolev inequality of order α, and ∇V is Hölder continuous of exponent s.

The third case interpolates between the log-Sobolev ($\alpha = 2$) and Poincaré ($\alpha = 1$) settings, and captures potentials V with tail growth $V(x) \approx \|x\|^\alpha$ as $\|x\| \to \infty$.

In case *(i)*, our result resolves an open question of Vempala and Wibisono (2019) on the Rényi bias of LMC, whereas our result in case *(iii)* resolves the open question of obtaining a sampling guarantee under the sole assumption of a Poincaré inequality and a smoothness condition. In all cases mentioned above, our results improve upon the state-of-the-art bounds for LMC.

The main technical contribution of this work is to introduce techniques for bounding error terms under a certain change of measure, which is a new feature in Rényi analysis. In cases *(i)* and *(ii)*, we apply the interpolation of Vempala and Wibisono (2019) and handle the change of measure using the Donsker–Varadhan variational principle and the log-Sobolev inequality. This approach fails for case *(iii)*, so we develop a novel technique based on establishing sub-Gaussianity of the LMC iterates via a change of measure principle, and then subsequently applying Girsanov's theorem.[1]

Keywords: Langevin Monte Carlo, Latała–Oleszkiewicz inequality, modified log-Sobolev inequality, Poincaré inequality, Rényi divergence.

1. Extended abstract. Full version appears as [arXiv:2112.12662, v1].

References

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