

# Supplementary material for mmVAE: multimorbidity clustering using Relaxed Bernoulli $\beta$ -Variational Autoencoders

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Primary condition		Prevalence (of $N = 290812$ )	Total co-morbidities		
			1	2	3+
Mental health	Depression	133060 (45.78%)	41929	37533	53628
	Anxiety	97239 (33.45%)	27634	27040	42581
	OthMental	47124 (16.2%)	11746	12153	23225
	eatingdisorderuom	11645 (4.0%)	2795	2816	6034
	substance misuse	11638 (4.0%)	2085	2559	6994
	SMHmm	11108 (3.8%)	1294	2002	7812
	alcoholproblem	5683 (2.0%)	9283626	1129	3626
Cardiovascular	hypertension	6876 (2.4%)	2113	1763	3000
	CongHeart	2770 (1.0%)	994	715	1061
	Valve	1115 (0.4%)	346	291	478
	stroke	1010 (0.3%)	266	263	481
	IHD MI	664 (0.2%)	156	144	364
	AF	219 (0.1%)	40	58	121
	Cardiomyopathy	207 (0.1%)	52	53	102
Dermatology	OthSkin	28085 (9.7%)	9231	7362	11492
	psoriasis mm	18620 (6.4%)	6428	4985	7207
	Atopic eczema	16581 (5.1%)	4430	4264	7887
Gynaecology	Female infertility	24121 (8.3%)	8893	6465	8767
	pcoskoo	21012 (7.2%)	6946	5639	8427
	endometriosis	10507 (3.6%)	3151	2729	4627
	leiomyoma	3338 (1.1%)	1205	846	1287
Haematology	VTEall	3851 (1.3%)	1112	902	1837
	perniciousanaemia	858 (0.3%)	231	237	390
	sickle cell	119 (0.3%)	46	37	36
	haemophilia	63 (0%)	22	16	25
Rheumatology	InflamArth	6609 (2.3%)	2145	1670	2794
	Ehler	1691 (0.6%)	440	416	835
	SpondArth	984 (0.3%)	232	240	512
Orthopaedic	chronicbackpain	3107 (1.1%)	735	756	1616
	scoliosis	3238 (1.1%)	1148	863	1227
	osteoporosis	571 (0.2%)	141	100	330
	Vertebrae	5885 (2.0%)	1672	1432	2781
Neurology	Migraine	64686 (22.2%)	21332	17406	25948
	OthHeadache	23626 (8.1%)	6333	6022	11271
	epilepsy mm	7238 (2.5%)	2346	1872	3020
	MS	900 (0.3%)	300	237	363
	spina bifida	644 (0.2%)	197	170	277
	iih	577 (0.2%)	144	140	293
	periph neuro	2776 (1.0%)	728	672	1376



Primary condition		Prevalence (of $N = 290812$ )	Total co-morbidities		
			1	2	3+
Respiratory	Asthma	85315 (29.3%)	30375	22920	32100
	PulmHtn	45 (0%)	11	9	25
	interstitiallungdiseasemm	50 (0%)	11	13	26
	bronchiectasisdraftv1	382 (0.1%)	75	93	214
	cf	239 (0.1%)	79	78	82
	osafinal	854 (0.3%)	250	177	427
	sarcoid	292 (0.1%)	92	78	122
	copd	323 (0.1%)	44	53	226
Other	Allergic Rhinoconjunctivitis	92340 (31.8%)	34050	24925	33429
	ibs mm	49439 (17.0%)	14435	13054	21950
	Thyroid	15824 (5.4%)	5327	4101	6396
	AutoSkin	4108 (1.4%)	1357	1121	1630
	InflamBowel	3348 (1.2%)	1157	918	1273
	InflamEye	3270 (1.1%)	1104	852	1314
	cholelithiasis	8682 (3.0%)	2579	2125	3978
	prithrombocytopenia imrd	578 (0.2%)	212	161	205
	Cancer	2890 (1.0%)	957	791	1142
	cataract	861 (0.3%)	269	214	378
	deaf	889 (0.3%)	278	243	368
	CKDall	604 (0.2%)	95	155	354
	ChrLiverAll	2446 (0.8%)	696	531	1219
	ulcer peptic	1562 (0.5%)	430	372	760
	slesystemic2019	612 (0.2%)	154	143	315
	oa	2204 (0.8%)	568	509	1127
	AdrenalAll	62 (0%)	19	9	34
	Pituitary	2010 (0.7%)	678	481	851
	retinal detach	473 (0.2%)	157	119	197
	pth	134 (0%)	37	30	67
	hfincidenceprevkoo	383 (0.1%)	81	91	211
	blindmm	118 (0%)	27	22	69
	solidorgantransplant	181 (0.1%)	26	38	117
	NeuroDev	2542 (0.9%)	724	632	1186
	turnerssyndrome imrd	57 (0%)	24	7	26
	marfansyndrome imrd	125 (0%)	32	35	58
	HIVall	267 (0.1%)	133	63	71
	DiabAll	5071 (1.7%)	1350	1243	2478
	DiabRetino	1720 (0.6%)	328	412	980
	coeliac	1529 (0.5%)	477	406	646
	urolithiasis	1973 (0.7%)	598	480	895
	Somatoform	5060 (1.7%)	269	214	378

Table 1: The total number of patients with each of the 79 health conditions in the full dataset (after removing non-multimorbidity previous pregnancy cases), and the total number of patients with different numbers of co-morbidities.

Conditions	Condition combination	Prevalence
2+	Depression, Anxiety	13329 (4.59%)
	Asthma, AllergicRhinoConj	9967 (3.43%)
	Depression, other mental health	4162 (1.43%)
	Asthma, Depression	3799 (1.3%)
	AllergicRhinoConj, Depression	3703 (1.3%)
	AllergicRhinoConj, Migraine	3432 (1.1%)
	Migraine, Depression	3163 (1.1%)
	Asthma, Migraine	2717 (0.9%)
	Anxiety, Depression, other mental health	2715 (0.9%)
	AllergicRhinConj, Anxiety	2253 (0.8%)
	AllergicRhinConj, Anxiety, Depression	2195 (0.8%)
3+	Anxiety, Depression, other mental health	2715 (0.9%)
	AllergicRhinConj, Anxiety, Depression	2195 (0.8%)
	Depression, Anxiety, Asthma	2113 (0.73%)
	Asthma, Anxiety, Depression	2113 (0.7%)
	Migraine, Anxiety, Depression	1863 (0.6%)
	Anxiety', Depression, ibs	1609 (0.6%)
4+	Asthma, AllergicRhinConj, Anxiety, Depression	931 (0.32%)
	Asthma, Anxiety, Depression, other mental health	510 (0.18%)
	AllergicRhinConj, Migraine, Anxiety, Depression	451 (0.16%)
	AllergicRhinConj, Anxiety, Depression, ibs	422 (0.2%)
	Asthma, Migraine, Anxiety, Depression	411 (0.1%)

Table 2: The top unique combinations of health conditions, and their respective prevalence for different levels of multimorbidity.

## 1. Experiments

In this section we show additional results which were excluded from the main text due to space constraints. We present the results for multiple choices of  $\beta$  on the reduced (i) asthma (ii) female infertility and (iii) cancer sub-group. In Fig. 1 we plot the factor odds ratios. In Fig. 2-3 we plot the prevalence plots for each of these factor. In Fig. 4-5 we plot the cluster odds ratios. In Fig. 6-7 we plot the prevalence plots for each of these clusters. In Fig. 8 we plot the cluster-factor association matrices. We observe a sharp decrease in factor-cluster sparsity as  $\beta$  increases.

In Fig. 9 we plot the *out-of-distribution* prevalence histograms. We observe that as  $\beta$  increases, and particularly as it shifts into a positive regime, the number of clusters out-of-distribution samples get mapped to overfits.

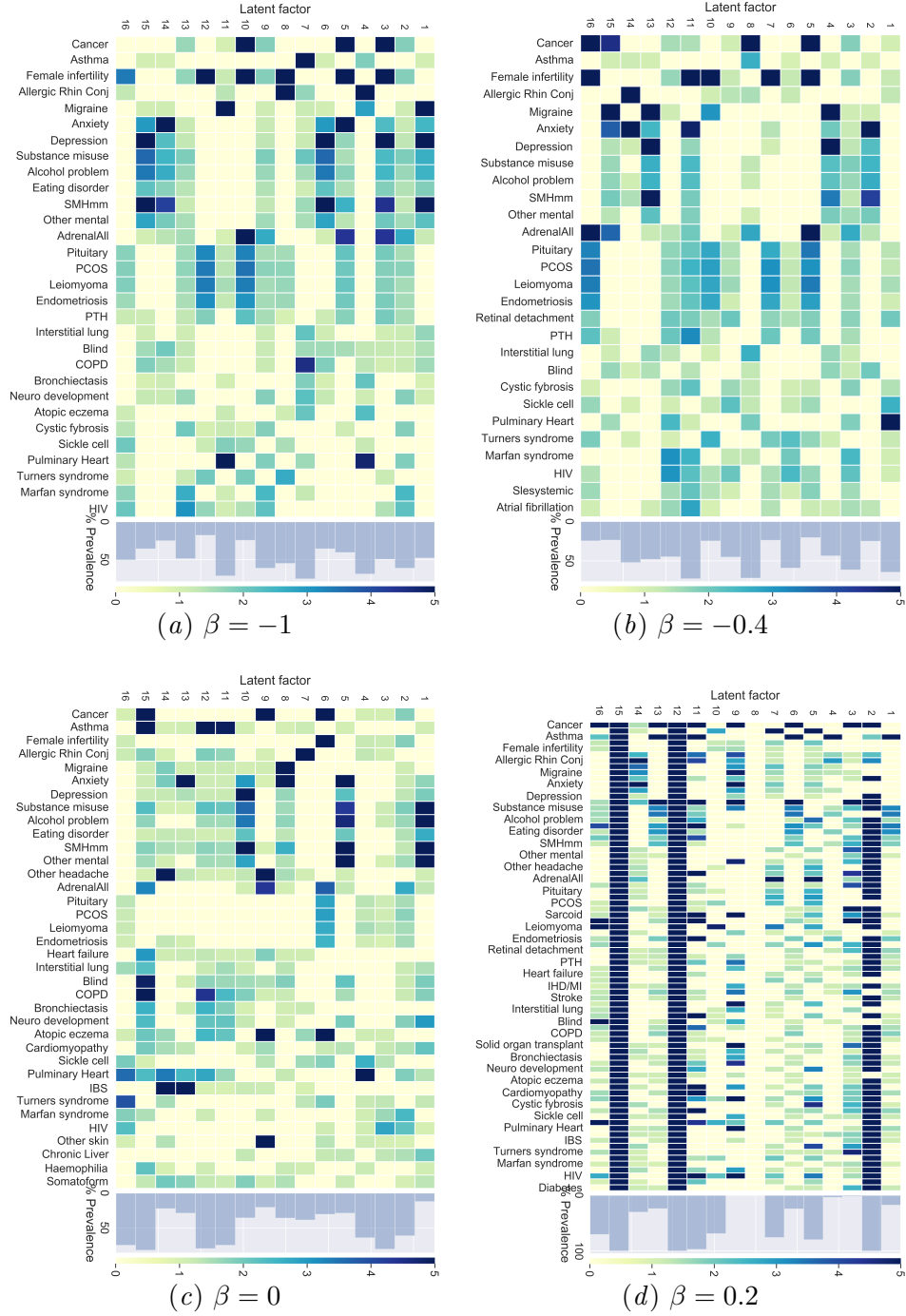


Figure 1: Factor odds ratio on a cohort with (i) asthma (ii) cancer and (iii) female infertility, and the percentage of individuals with each factor. Conditions with no odds ratio above 2 are filtered and odds ratios are truncated.

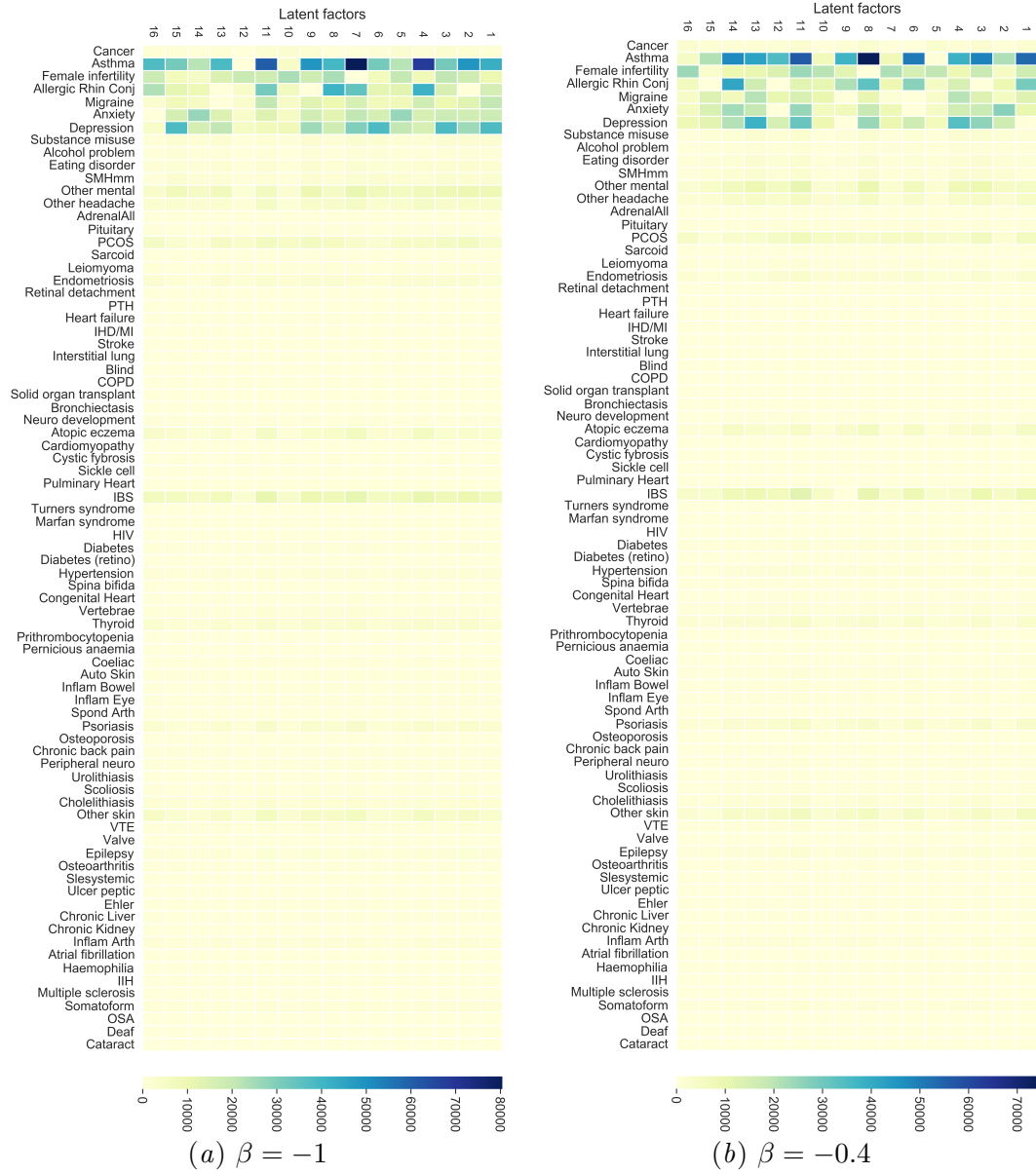


Figure 2: Factor prevalence on a cohort with (i) asthma, (ii) cancer, or (iii) female infertility health conditions.



Figure 3: Factor prevalence on a cohort with (i) asthma, (ii) cancer, or (iii) female infertility health conditions.



Figure 4: Clustering odds ratio on a cohort with (i) asthma, (ii) cancer, or (iii) female infertility health conditions, and the percentage of individuals in each cluster. Clusters with fewer than  $< 1\%$  of the samples are not shown and odds ratios are truncated.



Figure 5: Clustering odds ratio on a cohort with (i) asthma, (ii) cancer, or (iii) female infertility health conditions, and the percentage of individuals in each cluster. Clusters with fewer than  $< 1\%$  of the samples are not shown and odds ratios are truncated.



Figure 6: Cluster prevalence on a cohort with (i) asthma, (ii) cancer, or (iii) female infertility health conditions.





Figure 7: Cluster prevalence on a cohort with (i) asthma, (ii) cancer, or (iii) female infertility health conditions.

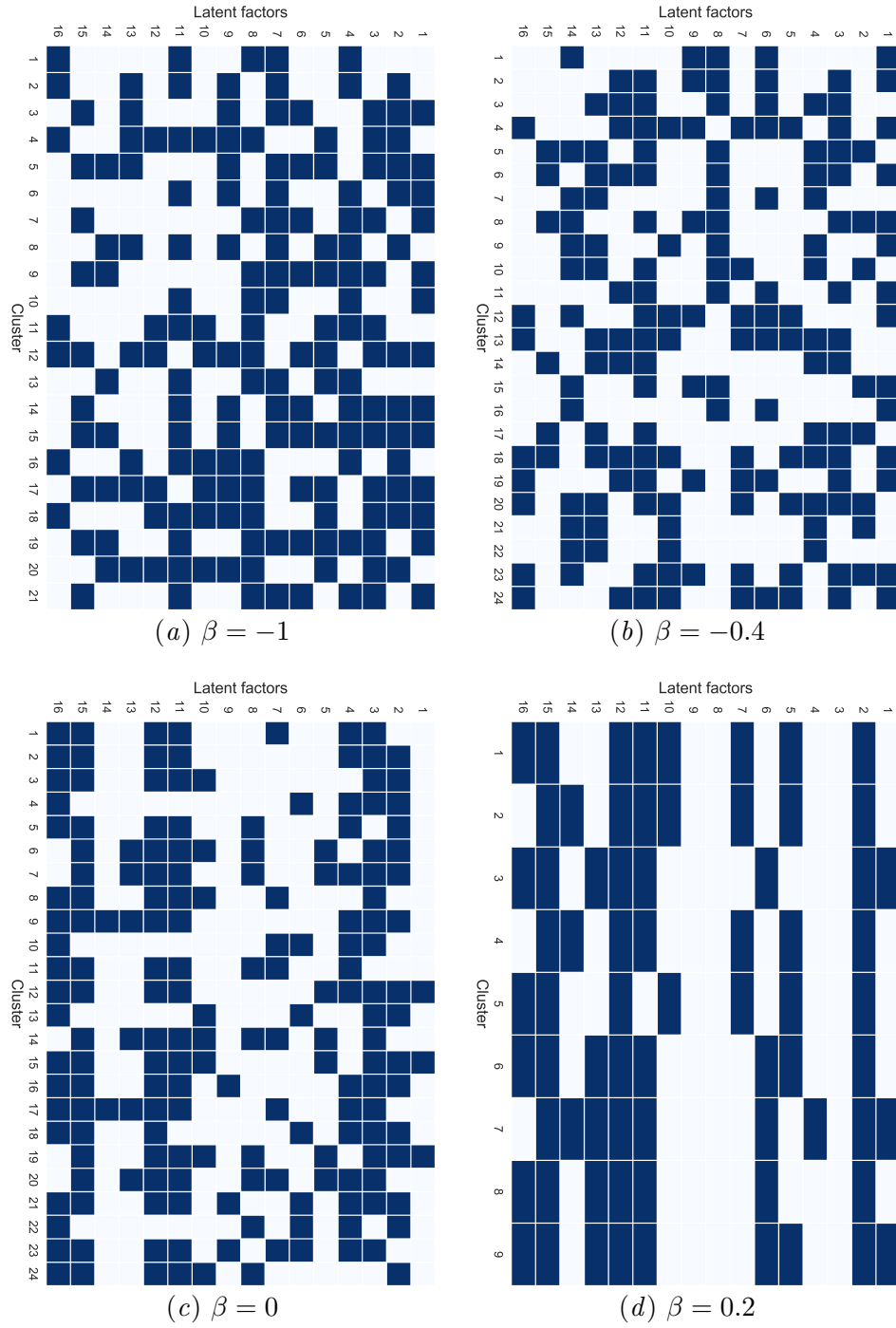


Figure 8: Cluster factor association matrices, indicating which factors are ‘turned on’ for each cluster.

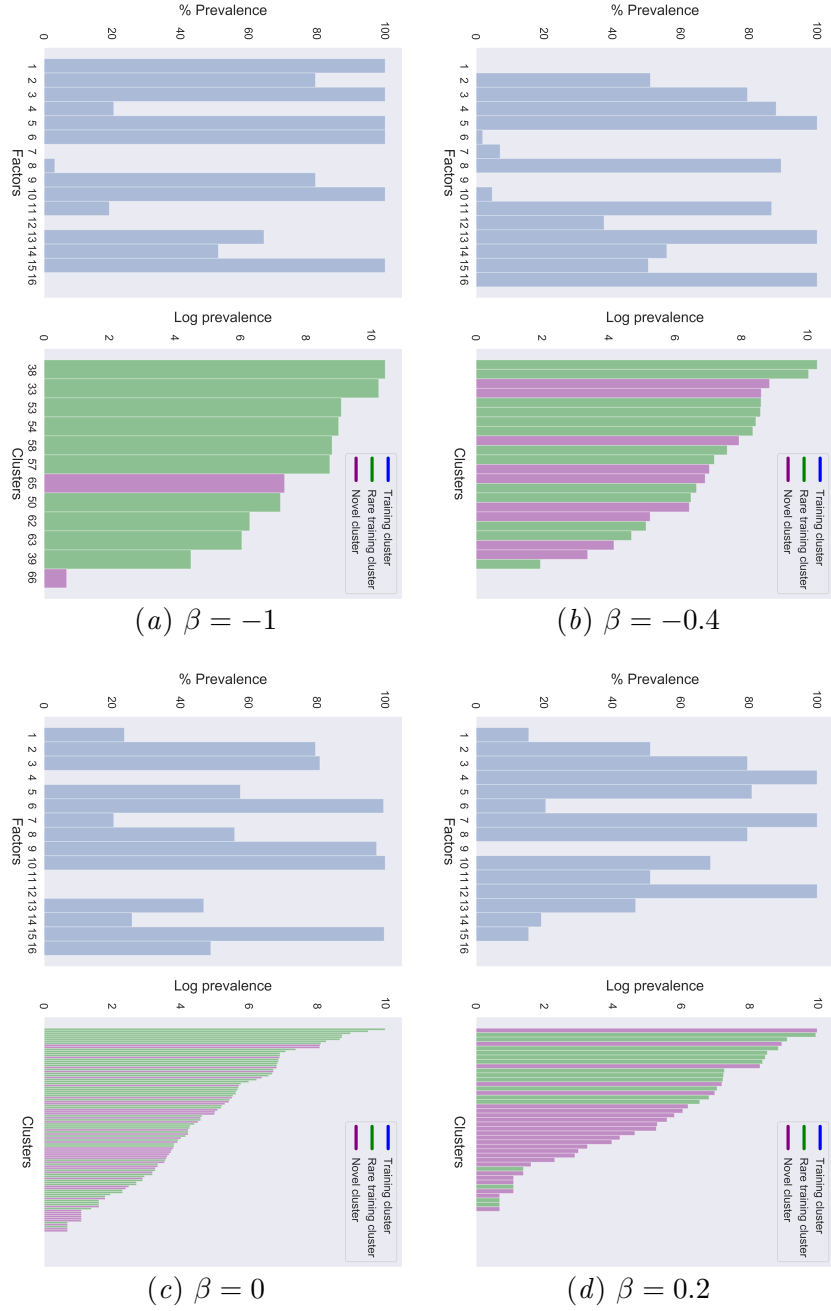


Figure 9: *Out-of-distribution* histogram plots. A test depression cohort is encoded using a mmVAE model trained on (i) asthma, (ii) cancer, or (iii) female infertility cases. No test combinations have been seen in training. Above (blue): The prevalence of test cases in each factor. Below (green/pink): The prevalence of test cases in clusters, labelled given their relation to the training clusters.