PREDICTIVE LEARNING ENABLES NEURAL NETWORKS TO LEARN COMPLEX WORKING MEMORY TASKS

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ABSTRACT

Brains are thought to engage in predictive learning - learning to predict upcoming stimuli - to construct an internal model of their environment. This is especially notable for spatial navigation, as first described by Tolman’s latent learning tasks. However, predictive learning has also been observed in sensory cortex, in settings unrelated to spatial navigation. Apart from normative frameworks such as active inference or efficient coding, what could be the utility of learning to predict the patterns of occurrence of correlated stimuli? Here we show that prediction, and thereby the construction of an internal model of sequential stimuli, can bootstrap the learning process of a working memory task in a recurrent neural network. We implemented predictive learning alongside working memory match-tasks, and networks emerged to solve the prediction task first by encoding information across time to predict upcoming stimuli, and then eavesdropped on this solution to solve the matching task. Eavesdropping was most beneficial when neural resources were limited. Hence, predictive learning acts as a general neural mechanism to learn to store sensory information that can later be essential for working memory tasks.

1 INTRODUCTION

Animals learn to create internal representations of their environment, even if these are not directly associated with attaining rewards (Tolman, 1948). How might an organism develop an internal model of their environment in the absence of reward? A candidate mechanism is predictive learning (Bar, 2009; Friston et al., 2016; Alexander & Brown, 2018; Nagai, 2019; Whittington et al., 2020; Recanatesi et al., 2021) which may arise even through passive exposure (Nagai, 2019). In predictive learning, animals continuously attempt to predict their future state, given their current state and actions. The utility of such models that predict future states has been extensively studied in reinforcement learning. In contrast to ‘model-free’ agents (that only reinforce actions that have previously led to rewards), ‘model-based’ agents can rapidly adapt to changes in the environment or reward structure, instead of starting from scratch (Daw et al., 2011; Russek et al., 2017; Wayne et al., 2018). Further, predictive learning in recurrent neural networks (RNNs) can extract the latent structure of the environment during a period of unrewarded exploration, actualizing the proposal that predictive learning acts as a helper function in the brain by encoding the environment (Whittington et al., 2020; Recanatesi et al., 2021).

Neuroscience experiments have revealed that passive learning of the environment may indeed encode spatial structure in the brain, as observed in the hippocampus (O’Keefe & Dostrovsky, 1971), but also nonspatial sequences of perceptual states, which can be detected even in primary sensory cortex (Libby & Buschman, 2021). However, in a purely passive learning paradigm, it remains unclear what benefit an organism would gain from actively maintaining predictions of upcoming sensory inputs. To address this question, we focus on a working memory task, which specifically requires maintaining information across time, and ask whether actively maintaining sensory predictions across a time delay can improve learning.

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We use artificial neural networks to show that predictive learning can give rise to an internal stimulus representation that enables networks to learn a more difficult working memory task. Specifically, in our experiments, small, sparse RNNs could not learn a delayed-match-to-category (DMC) task without simultaneously engaging in predictive learning. When RNNs were faced with these two tasks, they would first solve the prediction task, which created an internal stimulus representation that kick-started learning of the DMC task. This phenomenon - where two tasks depend on a latent feature that can only be learned from one of the tasks, thereby initiating learning of the other - has been termed ‘eavesdropping’ (Caruana, 1997), and constitutes a form of multitask learning that is known to boost learning performance in artificial networks (Bell & Renals, 2015; Yu & Jiang, 2016; Liu et al., 2020). Our results suggest that neural systems could engage in eavesdropping by using predictive learning as helper task.

We also show that predictability does not need to persist constantly, in order to produce long-term benefits. It was sufficient to introduce stimulus predictability only transiently, making simulated annealing an effective mechanism to kick-start the learning of complex concepts. This finding aligns with the dynamic nature of natural environments, where stimulus correlations come and go. Finally, we leverage our results to formulate testable predictions for experimental neuroscience.
Figure 2: **Quantification of the eavesdropping effect**

A) Speed of convergence of the prediction task as a function of network size and sparsity. B) The eavesdropping effect shown in Fig. 1E was quantified by measuring the difference in final loss (averaged over 5 final epochs), confirming that only dual task networks could learn the matching task. C) The magnitude of the eavesdropping effect depended on the number of nonzero connections (defined by $|W| > 0.01$, result is robust to different thresholds). The number of nonzero connections was varied by parametrically changing the sparsity regularization parameter $\lambda$ of Eq. 3 from 0 to 0.1 and measuring the fraction of nonzero connections. D) The prediction task was solved equally well when $S_2$ was delivered later, during the GO window. E) Eavesdropping only occurred when the stimuli were predictable ($P(\alpha = \beta) = 0.75$), as in prior panels. Panels B, D and E used a two-sided Wilcoxon signed-rank test to assess significance ($P$ values shown). All panels show the mean ± 95% confidence interval of 10 simulated networks per data point for panel A and 20 simulated networks per data point of all other panels (error bars or shaded area).

2 **RESULTS**

2.1 **RECURRENT NEURAL NETWORKS USE EAVESDROPPING TO BOOTSTRAP LEARNING**

We trained standard recurrent neural networks (RNNs) to solve two working-memory tasks, either individually or simultaneously (Fig. 1A). Trials consisted of a sequence of two stimuli and a GO cue, interleaved by delay periods. The prediction task asked networks to continuously predict the next upcoming input stimuli. The matching task prompted during the GO period whether the two previously presented stimuli matched (Fig. 1B). Importantly, the stimuli were predictable, or biased, because a match occurred on 75% of trials. This means that the first stimulus $S_1$ was predictive of the second stimulus $S_2$ on most trials. Notably, networks could only learn the matching task when it was presented jointly with the prediction task (Fig. 1C, D, E). Here, networks first solved the prediction task, after which learning of the matching task ensued. As we will detail in the next sections, this happened because the prediction task unveiled a latent feature - the memory of the first stimulus $S_1$ - which was required to solve the matching task. This phenomenon, in which two tasks share a feature that can (easily) be learned from one, thus catalyzing learning in the second, is a multitask learning mechanism called ‘eavesdropping’ (Caruana, 1997).

2.2 **QUANTIFICATION OF THE EAVESDROPPING EFFECT**

To quantify the eavesdropping effect (Fig. 1E) and explore what conditions facilitated this learning strategy, we first optimized the number of RNN hidden units based on their performance on the prediction task alone (Fig. 2A). We
found that networks with 20 neurons converged on the prediction task with near-maximal speed of convergence. We then assessed the eavesdropping effect by calculating the difference in performance (i.e., their final loss value after training) between matching single task networks (Fig. 1C, E, in orange) and dual task networks (Fig. 1C, E, in blue), averaged over 20 simulations of each (Fig. 2B).

Next, we quantified how the eavesdropping effect depended on the difficulty of the tasks, by varying the strength of sparsity regularization (Fig. 2C). We found that eavesdropping was most effective at a range of medium sparsity values from 30% to 80% non-zero connections. When sparsity regularization was too strong (i.e., less than 25% of all possible connections were allowed), both learning paradigms failed to learn. When sparsity regularization was very weak (i.e., more than 80% of all connections were allowed) both paradigms partially converged. Hence, the effectiveness of eavesdropping depended on the difficulty of the tasks, and eavesdropping allowed sparse networks to learn the matching task which is typically beneficial to prevent overfitting. Correspondingly, the best absolute performance on the matching task was only achieved with sparse networks that used eavesdropping (Fig. 2C). We performed this analysis across a range of network sizes \( N \), which resulted in an equivalent range of sparsity values where networks could only learn the matching task with eavesdropping (Fig. S1). For the remainder of this study, we used the optimal fraction of nonzero connections (77%) from Fig. 2C.

Networks had to create an internal representation, i.e., a memory, of \( S_1 \) to predict the upcoming stimulus \( S_2 \). For the matching task, networks also had to memorize \( S_1 \) (to compare the two stimuli), but for a longer period of time (to compare with \( S_2 \)). Therefore, the difference in performance (Fig. 2B) could in theory be explained by vanishing gradients. However, we found that this is not the case, as the prediction task was equally well solved when the delay period between \( S_1 \) and \( S_2 \) was increased to mirror the matching task (Fig. 2D).

Finally, as expected, we found that stimulus predictability \( P(\alpha = \beta) > 0.5 \) was crucial for eavesdropping (on the prediction task) to occur: dual task networks failed to learn the matching task when they were trained with uncorrelated stimuli (Fig. 2E).

We have shown that RNNs solved the matching task by eavesdropping on the solution of the prediction task. We have purposefully used a small and simple network, and have optimized the parameters that give rise to eavesdropping (Fig. 2). This now enables us to pinpoint why learning the prediction task catalyzed learning of the matching task.

### 2.3 Matching Task Eavesdrops on the Stimulus Memory of the Prediction Task

Dual task networks gradually enhanced their neural representation of the \( S_1 \) stimulus over the course of training (Fig. 3A). First, the prediction task induced a memory of \( S_1 \) until the \( S_2 \) presentation (epoch 8, Fig. 3A), which is subsequently strengthened to facilitate the matching computation (Fig. 3B). We define the memory of stimulus \( S_1 \) at time \( t \) as the decoding accuracy of \( S_1 \) of the differentiated neural activity at that time (see Fig. 3A), evaluated across trials using logistic regression (see Methods). Fig. 3C shows how the memory of \( S_1 \) dynamically evolved for all three network types after training. Matching single task networks (orange) failed to converge, as they did not create a memory of \( S_1 \) beyond the time when \( S_1 \) was presented to the networks. However, prediction-only networks (red) created a memory representation of \( S_1 \) and maintained this at perfect accuracy until the \( S_2 \) presentation, as expected (because the optimal solution would use \( S_1 \) information to predict \( S_2 \)). Dual task networks (blue) also maintained a perfect memory of \( S_1 \) after convergence that slightly extends the duration of prediction-only networks, so that \( S_1 \) and \( S_2 \) could be compared to perform the matching task.

The prediction task and matching task thus share the same latent feature: \( S_1 \) memory. The prediction task was solved by creating a memory of \( S_1 \), on which dual-task networks could subsequently eavesdrop to solve the matching task (Fig. 3D). Conversely, prediction single task networks that are trained on uncorrelated data \( P(\alpha = \beta) = 0.5 \) do not require a memory of \( S_1 \) to achieve optimal performance, because \( S_1 \) is no longer predictive of \( S_2 \). Indeed we found that no \( S_1 \) memory was created in this case (Fig. 3F, red), which subsequently failed to bootstrap the matching task for the dual task learning networks (Fig. 3G, H). Hence, only networks trained on predictable stimuli created a memory of \( S_1 \) when solving the prediction task, and this facilitated learning of the matching task.

We next investigated how the neural representation of \( S_1 \) (as shown in Fig. 3A) dynamically evolved during a trial, which is known to be variable network property (Orhan & Ma, 2019). Intriguingly, we found that dual task networks could embed several different solutions to achieve optimal performance, depending on the stability of the neural representation of \( S_1 \). When the subpopulation of neurons that encoded \( S_1 \) changed over the course of a trial, this gave rise to a cross-temporal anti-correlation, while an unchanged representation induced a cross-temporally correlated code (Fig. 3I). Finally, a mix of these neuron types decorrelated the memory code. In other words, the cross-temporal correlation of the \( S_1 \) representation depended on how variable the identity of its neurons was over time. We quantified
Figure 3: Matching task eavesdrops on the stimulus memory of the prediction task

A) The neural representation of $S_1$ is shown for an example network for 4 different epochs during training. Each raster plots shows the differentiated activity between $\alpha = 1$ and $\alpha = 2$ trials of all neurons (clipped between -1 and 1 for clarity). B) Similarly, the neural representation of the Match/Non-Match (M/NM) trial type is shown at the same 4 epochs. C) The memory of $S_1$ was determined per time point by decoding the stimulus identity from the network activity. Panels C-H show the average ± std across 20 simulations. D) Dual task networks gradually learned to enhance the memory of $S_1$ (different epochs are shown from light to dark blue). The 4 epochs of panels A and B are indicated in the figure. E) Dual task networks memorize $S_1$ until the $S_2$ stimulus is presented, after which they represent the Match/Non-Match identity of the trial. F, G, H) Equivalent figures for networks that were trained with uncorrelated stimuli. The prediction task no longer induces a $S_1$ memory (red), because it is not predictive of $S_2$ anymore. It subsequently fails to bootstrap the matching task in the dual task networks (blue), because the $S_1$ memory is lacking. I) The cross-correlation of the differentiated activity (as in panel A) is shown for 3 example networks that show anti-correlated, decorrelated and correlated coding respectively (quantified by the $S_1$-$S_2$ cross-correlation). J) The distribution of $S_1$-$S_2$ cross-correlation values of the 20 network simulations (as determined by the average of the 4 cross-temporal data points with $S_1$ vs $S_2$ from panel I).
Figure 4: Increasing matching task complexity reduces eavesdropping effect

A) We investigated the benefit of eavesdropping for 6 different tasks (including the task of previous figures). Each task is a combination of the following 3 conditions: Input domain: stimuli $S_1$ and $S_2$ either used the same input domain ($A_\alpha$ and $A_\beta$, i.e., delayed-match-to-sample (DMS) task) or different domains ($A_\alpha$ and $B_\beta$, i.e., delayed-match-to-category (DMC) task). Number of samples: each stimulus could either be chosen from 2 samples ($\{A, B\} \in \{(1, 0), (0, 1)\}$) or 4 samples ($\{A, B\} \in \{(\cos \theta, \sin \theta); \theta \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}\}$). Because these stimuli can also take negative values, a different output non-linearity (tanh instead of softmax) and loss function (mean least squares instead of cross-entropy) was used for the prediction task. (We verified that the 2-sample prediction task also converged when optimized with this output non-linearity and loss function.) Type of matching task: For the 4-sample condition, we considered a regular matching task ($A_\alpha = B_\beta$ in the case of different input domains), and a rotated matching task ($A_\alpha = B_\beta + \frac{\pi}{2}$).

B) The effect of eavesdropping was quantified for the 6 different matching tasks, by summing the sparsity-dependent loss of the matching task of Fig. S2 (i.e., the orange and blue lines, experiment was repeated for all tasks, with 20 networks per sparsity value). As the matching task complexity grew, eavesdropping became less effective because the dual-task networks became worse at solving the task.

2.4 Increasing matching task complexity reduces eavesdropping effect

Eavesdropping allowed RNNs to learn the matching task by utilizing memories from solving the prediction task. Consequently, eavesdropping failed when stimuli were not predictable (Fig. 3F-H), i.e., when $S_1$ was not predictive of $S_2$ such that the prediction task was no longer sufficiently informative of the matching task. Next we asked if we could similarly identify the limits of when eavesdropping is an efficient learning strategy by increasing the difficulty of the matching task instead.

We changed the matching task by considering 6 different combinations of three possible conditions: input domain, number of samples and matching task type (Fig. 4A). We then quantified the eavesdropping effect across sparsity values (as in Fig. 2C and Fig. S2) and summed over data points to obtain a single metric. In short, we found that as the matching task complexity increased, eavesdropping gradually became less effective because the dual-task networks could no longer solve the matching tasks. This decrease in eavesdropping efficiency was not caused by a failure of the networks to solve the prediction task, as networks still solved the prediction task at all task complexities and sparsity values (except for the two strongest sparsity values that lead to $\sim 0\%$ nonzero weights), see Fig. S3. Task performance was similar for both input domains (match-to-sample or match-to-category tasks), but decreased when more stimuli or a rotated-match task were introduced (Fig. 4B). Hence, as the difficulty of the matching task increases, eavesdropping on the solution of the prediction task becomes less useful for learning the matching task.

2.5 Simulated annealing enables RNNs to learn the matching task with uncorrelated stimuli

In the previous sections we have shown that eavesdropping only worked with predictable stimuli, because only then a memory of $S_1$ was created. To prove that the stimulus $S_2$ predictability - and the thereby induced $S_1$ memory -
Simulated annealing of stimulus predictability $P(\alpha = \beta)$ enables RNNs to learn the matching task. Whereas tasks with a constant $P(\alpha = \beta) = 0.5$ do not learn to solve the task.

Simulated annealing enables RNNs to learn the matching task with uncorrelated stimuli A) Dual task networks are able to learn the matching task with uncorrelated stimuli, by temporarily increasing and decreasing the stimulus $S_2$ predictability $P(\alpha = \beta)$ from 0.5 to 0.75 and back to 0.5. This simulated annealing of the stimulus predictability causes the prediction task to learn the memory of $S_1$, which then bootstraps learning of the matching task. Networks are able to maintain their stable solution of the matching task - which does not depend on stimulus $S_2$ predictability - when the stimuli are uncorrelated again. The end result is a network that has learned the matching task with uncorrelated stimuli.

B) In contrast, dual task networks are not able to learn the matching task with uncorrelated stimuli without simulated annealing (also see Fig. 2E). Both panels show the mean ± std of 10 simulated networks per network type.

only helped to initiate the learning process of the matching task, but was not necessary for performing the task after learning converges, we consider the following: we dynamically varied the stimulus $S_2$ predictability $P(\alpha = \beta)$ from a plateau of 0.5 to a plateau of 0.75 and back again to 0.5. We name this procedure simulated annealing, after the commonly-used technique in Monte Carlo sampling (MacKay, 2003, Ch. 30).

We found that dual task networks that engaged in this setting first learned to solve the prediction task using uncorrelated stimuli, and then gradually learned to memorize $S_1$ for predicting $S_2$ as stimulus predictability increased. This $S_1$ memory could then bootstrap learning of the matching task, and networks maintained a stable solution after the stimuli decorrelated again (Fig. 5A). Hence, the matching task could be performed with uncorrelated stimuli, but only by simulated annealing of the stimulus $S_2$ predictability, as networks with constant $P(\alpha = \beta) = 0.5$ did not converge (Fig. 5B). In summary, a transient period where stimuli are predictable provides the bootstrapping required to learn the demanding matching task, by inducing stimulus memories in the network activity.

3 DISCUSSION

3.1 SUMMARY AND LIMITATIONS

We have shown that predictive learning acted as a helper task for sparse RNNs to achieve optimal performance in a delayed-match-to-category (DMC) task. This was facilitated by eavesdropping, a multitask learning mechanism whereby the easier prediction task created a persistent neural representation of the stimulus that catalyzed learning of the matching task. Crucially, it was this stimulus representation (or memory) that was necessary for the matching task to initiate learning. Our results show that predictive learning can induce this representation, and was therefore sufficient to initiate learning of the matching task (Fig. 3).

Eavesdropping benefits were specific to certain parameters and task variants. The sparsity optimization showed that eavesdropping is effective for a range of parameter settings that ensure the matching task is neither too difficult (strong sparsity regularization) nor too easy (no sparsity regularization) (Fig. 2). Further, as the matching task complexity increased, networks failed to converge, even with eavesdropping (Fig. 4). Lastly, previous studies have optimized very similar matching tasks (including a variant of the rotated match task) without eavesdropping, using similar RNNs of 100-500 hidden units (Yang et al., 2019; Masse et al., 2019; Orhan & Ma, 2019). Our results show that although smaller networks naturally fail to learn these memory tasks, eavesdropping is able to facilitate success.

Small, interpretable neural networks are crucial for understanding learning mechanisms (Rudin, 2019), and can provide a good approximation of larger biological networks (Barak, 2017). Previous studies have shown that larger networks with more complex architectures can use predictive learning to boost performance in more difficult tasks, such as 3D...
navigation (Wayne et al., 2018), but their underlying mechanisms are difficult to dissect. After we confirmed that task performance was equivalent across a range of network sizes (Fig. S1), we adhered to a minimal working example of predictive learning, which enabled us to uncover eavesdropping as its facilitating mechanism (Fig. 3). Notably, the stimulus-prediction and DMC tasks that we considered are typical of controlled experiments performed in animals and humans (Freedman & Assad, 2006; Wan et al., 2020; Mohan et al., 2021; Libby & Buschman, 2021; Zhou et al., 2021), which allows us to use this theoretical study to make experimental predictions that can readily be tested in vivo. We will do so at the end of the Discussion section. Hence, crucially, the aim of this paper was not to develop a new Machine Learning heuristic, but rather to use artificial networks to further develop our understanding of biological learning.

3.2 PREDICTIVE LEARNING AND EAVESDROPPING IN NATURAL LEARNING

Eavesdropping is a form of multitask learning where two (or more) tasks that are optimized simultaneously share a latent feature that can only (easily) be learned in one of the tasks, hence catalyzing learning in the other task (Caruana, 1997). Eavesdropping, and multitask learning more generally, is conceptually related to both transfer learning and curriculum learning, which both use ‘helper’ tasks to improve learning. All these learning strategies have been proposed to occur naturally in animals (Elman, 1993; Ben-David & Schuller, 2003; Bengio et al., 2009; Weiss et al., 2016; Hassabis et al., 2017). However, it remains unclear what the helper task of eavesdropping could be in neural systems.

Simultaneously, predictive learning has been shown to benefit reinforcement learning (RL) agents (Russek et al., 2017; Wayne et al., 2018). In particular, the successor representation - a form of predictive learning (Dayan, 1993; Gershman et al., 2012) - has been able to provide a link between model-free and model-based RL (Russek et al., 2017), which are both known to occur in the brain (Daw et al., 2011).

In neuroscience, brains are thought to continuously engage in predictive learning, often motivated by normative frameworks (Rao & Ballard, 1999; Friston et al., 2016). Our study builds on this, by proposing an additional function of predicting sensory inputs. We have linked eavesdropping and predictive learning, by showing that predictive learning was sufficient to induce eavesdropping in environments where stimuli are correlated in time. Other learning processes that create stimulus representations, such as solving a delayed-response task (Yang et al., 2019; Duncker et al., 2020), could potentially also facilitate eavesdropping. However, in this work we have focused on predictive learning, given its prevalence in neuroscience.

Prediction may be useful as sensory correlations are ubiquitous. Our study suggests that prediction functions as a natural prerequisite for the ability to remember and process sensory experiences over time. Importantly, this does not entail that ongoing predictability is necessary to maintain a memory: In Fig. 5 we show that transient predictability is already enough to bring the system into a new minimum, which remains stable after stimulus predictability disappears. Thus, brief moments during which a complex task can be deconstructed into smaller parts can reveal a path to the solution, after which these components can be disregarded again (also see Knoblich et al. (1999)).

3.3 EXPERIMENTAL PREDICTIONS

Libby & Buschman (2021) found that mice actively encoded stimulus predictions during a passive learning paradigm (i.e., with no reward structure). Our study suggests that this learning is functionally important: representing stimulus correlations that are not directly relevant for behavior might still be a rewarding long-term strategy. Specifically, predictive learning can act as a module that facilitates learning of complex future task structures.

Taking these experimental findings together with our results, we propose the following hypothesis. Animals that have to solve a complex working memory task, such as DMC, should benefit from prior passive exposure to correlated sequences of the stimuli (that are later used in the DMC task). With no requirement for reward, they should internally attempt to predict these stimuli, thereby creating neural representations of the stimuli that can later accelerate learning. Prior exposure to uncorrelated sequences, in contrast, should not benefit learning, as random sequences cannot be predicted.

4 METHODS

Software & code availability All data was analyzed and visualized with Python 3.7, using PyTorch as automatic differentiation package, and various other standard libraries for analysis and visualization (Numpy, Scipy, Sciikit-Learn, Pandas, Matplotlib, Seaborn, IPython). All experiments and analysis were done on a 16-thread CPU of a desktop computer. All code and trained network simulations are available at: github.com/vdplasthijs/eavesdropping
Terminology  Vectors are denoted by bold notation (\( \mathbf{x} \)) and matrices by capital notation (\( X \)). Time is denoted by subscript \( t \), trials are denoted by subscript \( k \).

Tasks  We considered two tasks in Figs. 1, 2, 3, 5: first, the prediction task required networks to continuously predict the next upcoming input stimulus, second, the matching task required networks to report whether two stimuli matched at the end of each trial (Fig. 1A).

In Fig. 4 we considered the task described above, and 5 additional variants, by changing 1) the input domain, 2) the number of possible stimulus samples and 3) a direct match or rotated match. These are further described in the caption of Fig. 4.

Synthetic data  Each trial consists of a sequence of 2 stimuli \( S_1 \) and \( S_2 \) and a GO cue, interleaved by delay periods (Fig. 1B). Stimuli are one-hot vectors, with \( S_1 = A_\alpha \in \{ A_1, A_2 \} \) and \( S_2 = B_\beta \in \{ B_1, B_2 \} \) (see Fig. 1B). Hence, the total trial sequence is:

\[
\text{trial sequence} = (0, 0, A_\alpha, A_\alpha, 0, 0, B_\beta, B_\beta, 0, 0, \text{GO, GO, GO, GO})
\]

where each element of Eq. 1 is a one-hot vector. Networks are either trained on predictable sequences (\( P(\alpha = \beta) = 0.75 \)), or unpredictable (uncorrelated) sequences (\( P(\alpha = \beta) = 0.5 \)). The duration of all stimuli, GO and the delay periods is 2 time points. This ensures that networks that are trained on predictable sequences will encode both stimuli for optimal performance: \( A_\alpha \) information is used to predict the first \( B_\beta \) with 75% accuracy, and then the first \( B_\beta \) is used to predict the second \( B_\beta \) with 100% accuracy.

The network input \( \mathbf{x} \) is defined by the one-hot vectors of Eq. 1 (Fig. 1B) with added white noise (to prevent networks from overfitting) with standard deviation \( \sigma_x = 0.15 \) (we found numerically that the results were robust to variations in \( \sigma_x \)). For each network, we randomly generated 1000 trials, which are stratified-split into an 80/20 train/test ratio of trials. All results were presented on test data only.

Recurrent neural network training  We used standard recurrent neural networks (RNNs), consisting of 1 hidden layer of \( N = 20 \) neurons that all receive input from 6 input neurons and project to 8 output neurons (Fig. 1A). The (rate) activity of neurons \( r_t \) is defined by:

\[
\mathbf{r}_t = \tanh (\mathbf{U} \cdot \mathbf{x}_t + \mathbf{W} \cdot \mathbf{r}_{t-1} + \mathbf{b}_W)
\]

where \( \mathbf{x}_t \) is the input at time \( t \), \( \mathbf{b} \) is the learned bias, \( \mathbf{U} \) is the input weight matrix and \( \mathbf{W} \) the recurrent weight matrix. At the start of each trial, \( \mathbf{r} \) is initialized with \( r_{-1} \sim \mathcal{N}(0, 0.1 \cdot \mathbf{I}) \).

The 6-element prediction task output vector \( \mathbf{y}^{\text{pred}}_t \) that estimates the next stimulus is given by a softmax function:

\[
\mathbf{y}^{\text{pred}}_t = \text{softmax}(\mathbf{V} \cdot \mathbf{r}_t + \mathbf{b}_V)
\]

where \( \mathbf{V} \) is the prediction output weight matrix. Hence, \( \sum_{i=1}^{6} y^{\text{pred}}_t = 1 \), and the network predicts the probability of each possible outcome, rather than one deterministic value. Similarly, the 2 output neurons of the matching task \( \mathbf{y}^{\text{match}}_t \) must estimate \( \mathbf{y}^{\text{match}}_t = (1, 0)^T \) on Match (\( \alpha = \beta \)) trials and \( (0, 1)^T \) on Non-Match (\( \alpha \neq \beta \)) trials, and are defined by:

\[
\mathbf{y}^{\text{match}}_t = \text{softmax}(\text{ReLU}(T \cdot \mathbf{r}_t + \mathbf{b}_T))
\]

where \( T \) is the task output weight matrix. (Two output units instead of one were used to ensure networks actively had to report both possible outcomes, and an additional ReLU non-linearity was introduced to prevent the trivial solution \( T_M = -T_{\text{NM}} \).) The loss function \( \mathcal{L}_k \) of trial \( k \), used to train the model parameters \( \theta = \{ \mathbf{U}, \mathbf{V}, V, T, \mathbf{b} \} \), is the sum of the cross-entropy loss of either one or both tasks - depending on the network type - and a sparsity L1 regularization term:

\[
\mathcal{L}_k = \mathcal{L}^{\text{pred}}_k \cdot \mathbf{1}_{\text{pred}} + \mathcal{L}^{\text{match}}_k \cdot \mathbf{1}_{\text{match}} + \lambda \cdot \mathcal{L}^{\text{L1}}_k
\]

where \( \lambda \) is the sparsity regularization parameter (optimized in Fig. 2). During the training phase, the loss \( \mathcal{L}_k \) is accumulated during each trial \( k \). At the end of each trial, \( \mathcal{L}_k \) is backpropagated through time to update \( \theta = \{ \mathbf{U}, \mathbf{V}, V, T, \mathbf{b} \} \) using stochastic gradient descent (SGD) with a learning rate set to 0.002.

Quantifying stimulus representation strength  The strength of stimulus representations was quantified in Fig. 3C-H using the stimulus decoding accuracy of logistic regression. We trained decoders to decode the stimulus identity (e.g. \( S_1 \)) at each time point \( t \): \( P(\alpha = 1 | r_t) = 1 - P(\alpha = 2 | r_t) \). Decoders were trained across trials using logistic regression with L1 regularization (scikit-learn implementation with a SAGA solver, \( \text{max} \_ \text{iter} = 250 \) and \( C = 0.1 \), (Pedregosa et al., 2011)). To ensure that the stimulus predictability does not bias the decoders, we trained decoders with a new set of 800/200 train/test trials with uncorrelated stimuli \( P(\alpha = \beta) = 0.5 \). Decoding accuracy is subsequently defined as the mean probability of correctly decoding the stimulus identity.
AUTHOR CONTRIBUTIONS

All authors conceived the project and contributed to writing and editing of the manuscript. TLvdP performed all numerical analyses and wrote the manuscript draft. TPV and SGM supervised the project.

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Figure S1: **Eavesdropping effect dependency on network size.** The eavesdropping effect as in Fig. 2C was quantified for a range of network sizes $N$ (i.e., number of hidden units) from 5 to 100. The single task and dual task match losses are shown in panel A, and their differences in panel B. Network size is indicated by transparency (see legend on the right). Very small networks ($N = 5$) cannot solve the task, while larger networks generally require eavesdropping for optimal performance, especially when sparsity is constrained. The average ± 95% confidence interval of 50 simulations per data point is shown.

Figure S2: **Detailed breakdown of Fig. 4B.** Each panel is as in Fig. 2C, conducted for each of the 6 tasks as defined in Fig. 4. NB: For each task the same range of sparsity regularization parameter $\lambda$ values were evaluated, which could result in slight variations in number of nonzero connections (hence the variation of the $x$-axis positions of the data points across panels). The optimal loss is 0 for all tasks.
Figure S3: **The prediction task is solved for all tasks.** Each panel shows the prediction task loss of both the dual task networks and single (prediction) task networks, for each of the 6 tasks as defined in Fig. 4. Only networks with severe sparsity regularization, such that almost all connections were 0, failed to learn the task.
REFERENCES


