Proceedings of Machine Learning Research 204:1–3, 2023 Conformal and Probabilistic Prediction with Applications

# Online NoVaS Conformal Volatility Prediction

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## 1. Extended Abstract and Methodology

We propose an online approach for volatility forecasting that combines the NoVaS (Chen and Politis, 2020) studentization approach with the adaptive $\alpha$ adjustment of ACI (Gibbs and Candes, 2021). The primary advantage of the proposed approach is that it does not need to store historical data to compute empirical quantiles, thus making it efficient and scalable for real-time applications. Our focus is on realized volatility forecasting (McAleer and Medeiros, 2008). We use the absolute value of returns $r_t$ as proxy of daily realized volatility as $h_t := \sqrt{\frac{252\pi}{2}}|r_t|$ (Ederington and Guan, 2006). The proposed method studentizes the non-conformity scores $\{s_t := |h_t - \hat{h}_t|\}$ using an exponential NoVas methodology (Chen and Politis, 2020). The prediction intervals are constructed as:

$$C_t(\alpha_{t-1}) = \hat{h}_t \pm F_{t-1} q(1 - \alpha_{t-1}/2), \tag{1}$$

where $q(\cdot)$ is the quantile function of the standard normal distribution (justified by the studentization step) and $F_{t-1}$ is the studentization factor of $\{s_t\}$ as of $t-1$. Similarly as in (Gibbs and Candes, 2021) the $\alpha_{t-1}$ is updated as:

$$\alpha_t := \alpha_{t-1} + \gamma\left(\alpha - \mathrm{EMA}(\mathrm{err}_t, m_{t-1})\right), \tag{2}$$

where $\mathrm{EMA}(x, m)$ is the exponential moving average of $x$ with memory $m$, $\alpha$ is the target confidence, $m_{t-1}$ is the memory minimizer of the studentization criteria of the exponential NoVaS methodology, and $\mathrm{err}_t$ equals 1 if $h_t \notin C_t(\alpha_{t-1})$ and 0 otherwise. The proposed approach although non-parametric, assumes that minimizing excess kurtosis is enough to achieve normality of the studentized scores, this was corroborated on the data.

## 2. Experiments and results

All experiments where conducted with equal weighted daily returns of industry portfolios from 1926 to 2023 (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french`). We used a HAR based forecaster (Clements and Preve, 2021) calibrated with a rolling year OLS. As in (Bhatnagar et al., 2023), we report 100 times the average of:

**Coverage** $\left\{1 - \frac{1}{252}\sum_{\tau=t-251}^{t} \mathrm{err}_\tau \;:\; t > 252\right\}$,

**Error** $\left\{\max_{[t-251,t]} \left|\alpha - \frac{1}{252}\sum_{\tau=t-251}^{t} \mathrm{err}_\tau\right| \;:\; t > 252\right\}$,

**Width** $\{\mathrm{width}_t := \max C_t(\alpha_{t-1}) - \min C_t(\alpha_{t-1})\}$.

As a benchmark the ACI algorithm was used (Gibbs and Candes, 2021) with an exponential decay, of 120 days, weighting scheme (Barber et al., 2022) for the empirical quantiles. For both methods $\gamma = 0.2$, $\alpha = 0.2$ were used.

The average *Coverage* across all industries was 77.52 and 77.14 for ACI and NoVaS respectively, while the average *Width* across industries was 30.59 and 31.20 for ACI and NoVaS respectively. Detailed results per industry portfolio are presented in Table 1.

Table 1: Results with $\gamma = 0.2$ and $\alpha = 0.2$ from 1926 to 2023.

| | Coverage | | Error | | Width | |
|---|---|---|---|---|---|---|
| | ACI | NoVaS | ACI | NoVaS | ACI | NoVaS |
| NoDur | 78.146 | **79.450** | 3.633 | **2.913** | **22.286** | 26.069 |
| Durbl | 77.918 | **77.941** | **3.596** | 5.210 | **32.060** | 33.425 |
| Manuf | 78.198 | **79.322** | 3.439 | **3.049** | **28.144** | 30.851 |
| Enrgy | **74.154** | 66.095 | **8.256** | 19.332 | 50.350 | **37.773** |
| HiTec | **77.598** | 77.060 | **4.089** | 6.390 | 34.344 | **34.299** |
| Telcm | **77.095** | 75.898 | **4.700** | 8.612 | 35.829 | **35.324** |
| Shops | 78.128 | **79.459** | 3.515 | **3.399** | **27.415** | 30.606 |
| Hlth | **77.335** | 76.009 | **4.371** | 7.525 | 32.703 | **32.659** |
| Utils | 78.246 | **80.106** | 3.492 | **3.124** | **20.401** | 24.723 |
| Other | 78.387 | **80.031** | 3.236 | **3.008** | **22.343** | 26.264 |

## 3. Conclusions and Future Work

The proposed approach is comparable to ACI in terms of *Coverage* (see Figures 1 and 2 ) and *Width* with the extra potential benefit of needing a small bounded memory to operate. An analysis of why both analyzed algorithms did not perform with the energy portfolio (see 'Enrgy' in Table 1) is outstanding, as well as an analysis if the proposed approach translates to other financial forecasting use cases.

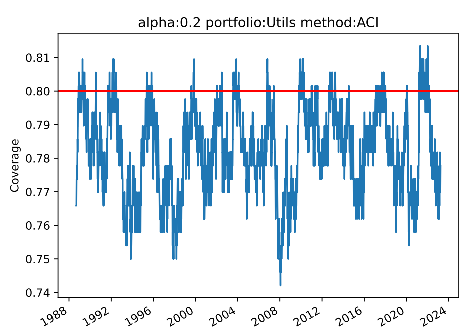


Figure 1: Rolling yearly coverage for ACI on Utils portfolio

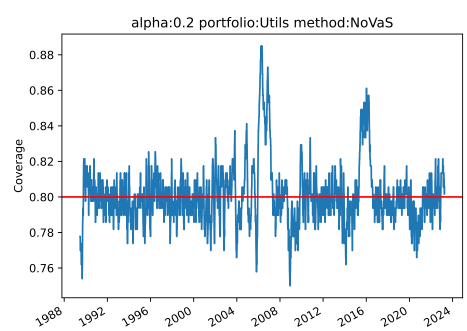


Figure 2: Rolling yearly coverage for NoVaS on Utils portfolio

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