A Comparison Between a Frequentist, Bayesian and Imprecise Bayesian Approach to Delay Time Maintenance

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Abstract

Delay time models are stochastic maintenance decision aid tools that divide the failure time of a system into the appearance of a defect and its evolution towards a breakdown. In this study, an imprecise Bayesian approach to delay time modelling has been developed and compared with the frequentist and precise Bayesian approach based on a virtual maintenance problem. The conditional failure rate was the unknown parameter that had to be estimated via eight samples of failure times of increasing size. The goal was to minimise two loss functions related to the downtime and cost. The frequentist and precise Bayesian methods converge towards the optimal decision as the sample size grows but are strongly sub-optimal when no or only few data are available. The imprecise Bayesian approach based on E-admissibility returns large decision intervals in the lack of data, thereby straightforwardly representing the crucial difference between knowledge and ignorance. Keywords: stochastic maintenance, robust Bayesianism, decision rules, frequentism

1. Introduction

In the stochastic modelling of maintenance, delay-time analysis (DTA) explicitly makes a difference between the appearance of a defect (at a time $U$) and the system breakdown caused by this defect at time $U + H$, where $H$ is the delay-time Christer (1999); Wang (2012a); Scarf et al. (2019). It can be seen as being a kind of periodic off-line condition monitoring. Christer and Waller (1984) developed the first detailed delay-time model back in 1984 in order to optimise either the downtime per unit time $D(T)$ or the cost per unit time $C(T)$ as an enterprise can be primarily interested in either of these two quantities. Delay time models are conceptually simple (thereby reducing the risk of over-fitting) and they can successfully be applied to a wide range of industrial problems Wang (2012a). In this work, we shall consider the simplest situation, namely that of a system with a single failure mode where a single defect appears at a random time and progressively evolves into a full-blown failure after a random and stochastically independent delay time. Such a model can be used to simulate multi-component systems where the failure of one component has no impact on the proper function of the other components and it was applied to the maintenance of three infusion-pump components Baker and Wang (1993) and to a sample of about 100 infusion pumps Baker and Wang (1991). DTA generally assumes that the time of appearance of a defect $U$ and the delay time between the arrival of that defect and the breakdown of the system $H$ follow two exponential or Weibull distributions. The parameters of these distributions are most often estimated via Maximum Likelihood Estimation (MLE) Baker and Wang (1991) but sometimes also through a classical Bayesian approach Wang and Jia (2007); Wang (2012b). There are not, however, yet any studies that represent our prior ignorance through a family of priors rather than through a single one, which is problematic as the reliance on a single prior can all too easily lead one to mistake ignorance for specific knowledge Norton (2010); Fischer (2019); Fischer and Vignes (2021).

The present work thus sets out to compare a frequentist, a classical Bayesian, and an imprecise Bayesian approach to the problem of DTA parameter estimation and inspection optimisation based on samples of failure times virtually generated via Monte-Carlo simulations. In Section 2, the stochastic model is laid out along with the samples that will be used for the parameter estimation. The frequentist, classical Bayesian and imprecise Bayesian approach are considered in Section 3, 4 and 5, respectively. The conclusions and an outlook are given in Section 6.

2. Delay Time Modelling and Problem Setting

2.1. Stochastic Decision Model

We consider a system that can at most have only one defect at the same time before a failure. We consider that both the appearance time of the defect $U$ and the delay time before the failure $H$ are exponentially distributed and we call their parameters $k_f$ (the appearance rate of defects in defects/month) and $\lambda_h$ (the conditional failure rate in failures/defect/month), respectively. It can be easily shown that the failure time $Y = U + H$ has a probability density function given by

\[
f_Y(y) = k_f \lambda_h e^{-k_f y} \int_{h=0}^{y} e^{(k_f-\lambda_h)h} dh
\]
where $\lambda_h \neq k_f$.

Let us now suppose that an inspection is carried out every $T$ units of operating time, i.e. time during which the machine is operational and isn’t being repaired or inspected. $T = +\infty$ means that there is no inspection. Let $d_{br}, d_i, d_{defect}$ be the downtime due to a breakdown, an inspection and an inspection repair (when a defect is detected), respectively. It can be interesting for an enterprise to minimise the ratio between the cumulated downtime and the cumulated operating time given that the inspection period is equal to $T$.

Since $Operating\ Time = \min(Y, T)$, by virtue of the law of total expectation, we have

$$
E(Operating\ Time) = E(Y | Y \leq T) F_Y(T) + T \left(1 - F_Y(T)\right)
$$

with $F_Y$ being the cumulative distribution function (cdf) of the failure time $Y$.

Let $X_{\text{intact}}, X_{\text{defect}}$ and $X_{\text{failure}}$ be binary random variables equal to 1 when there is no defect during the renewal cycle, one defect that does not lead to a failure and a failure, respectively. We have

$$
Downtime = X_{\text{intact}}d_i + X_{\text{defect}}(d_i + d_{defect}) + X_{\text{failure}}d_{br},
$$

so that

$$
E(Downtime) = p_{\text{intact}}d_i + p_{\text{defect}}(d_i + d_{defect}) + p_{\text{failure}}d_{br}.
$$

Furthermore,

$$
p_{\text{intact}} = 1 - F_U(T)
$$

and

$$
p_{\text{failure}} = F_Y(T).
$$

Likewise, let $c_{br}, c_i, c_{defect}$ be the cost due to a breakdown, an inspection and an inspection repair (when a defect is detected), respectively. It is interesting to minimise

$$
C_1(T) = \frac{E(Cost)}{E(Operating\ Time)}
$$

with

$$
E(Cost) = p_{\text{intact}}c_i + p_{\text{defect}}(c_i + c_{defect}) + p_{\text{failure}}c_{br}.
$$

An enterprise can be interested in minimising the cost by unit of operating time $C_1(T)$ but also the downtime by unit of operating time $D_1(T)$ (when too much downtime could lead to the loss of market shares, for example). Often times, a compromise between these two goals has to be made.

Let $T_{D_1,\text{opt, } S}$ be the optimal inspection period minimising $D_1(T)$ based on the information contained in a sample $S$ of failure times (when $S = \emptyset$, this would be a decision made in the absence of information other than the parameter bounds). $T_{D_1,\text{opt, } \lambda_h,\text{true}}$ is the real optimal inspection period based on the real value of $\lambda_h$. Let us consider that the enterprise will use this type of pumps in 6 factories for a period equal to $t = 4$ years of operational time (i.e. not including all the potential downtimes)\footnote{Of course, another number of factories can be considered according to the industrial situation.}; it is then interesting to compute the quantity

$$
\Delta Downtime = 6 \left(D_1(T_{D_1,\text{opt, } S}, \lambda_h,\text{true}) - D_1(T_{D_1,\text{opt, } \lambda_h,\text{true}}, \lambda_h,\text{true})\right) t
$$

which represents the sub-optimality of the decision made on the basis of sample $S$, i.e. the amount of downtime that would have been avoided if we had known the real parameter value perfectly. $D_1(T_{D_1,\text{opt, } S}, \lambda_h,\text{true})$ is the expected downtime based on the estimated optimal inspection period obtained thanks to sample $S$ whereas $D_1(T_{D_1,\text{opt, } \lambda_h,\text{true}}, \lambda_h,\text{true})$ is the expected downtime based on the true optimal inspection period determined with the true value of $\lambda_h$.

Likewise, we can define for the cost

$$
\Delta Cost = 6 \left(C_1(T_{C_1,\text{opt, } S}, \lambda_h,\text{true}) - C_1(T_{C_1,\text{opt, } \lambda_h,\text{true}}, \lambda_h,\text{true})\right) t
$$

which is the sub-optimality of the decision made on the basis of sample $S$ with respect to $C_1(T)$.\footnote{Of course, another number of factories can be considered according to the industrial situation.}
We consider a pump that can have at most one defect whose lengths are equal to 2, 5, 10, 20, 60, 200, 600, and 47.75 days and that for minimising the likelihood function which is given by Equation (11).

\[
L(\lambda_h | y_1, y_2, \ldots, y_n) = \prod_{i=1}^{n} f_y(y_i)
\]

(11)

There does not appear to be an analytical expression for the MLE. Instead, the likelihood function must be numerically maximised. Generally, the maximum likelihood estimate of the parameter(s) is used to minimise the loss functions \(C_1\) and \(D_1\) Baker and Wang (1991); Christer et al. (1995). According, the MLE corresponding to the different samples were used to compute \(D_{1,opt,S}\), \(C_{1,opt,S}\), \(\Delta D_{\text{downtime}}\), and \(\Delta C_{\text{cost}}\) (see Equation 9 and 10). The results can be seen in Table 1 (where the last line corresponds to the results obtained with the true parameter value).

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>(\lambda_{h,MLE})</th>
<th>(D_{1,opt,S})</th>
<th>(C_{1,opt,S})</th>
<th>(\Delta \text{Downtime})</th>
<th>(\Delta \text{Cost (€)})</th>
</tr>
</thead>
<tbody>
<tr>
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<td>67.7731</td>
<td>3.2699</td>
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<td>0.1278</td>
<td>49.4582</td>
</tr>
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<td>46.1062</td>
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<td>0.0766</td>
<td>29.7284</td>
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<td>60</td>
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<td>49.3381</td>
<td>51.9885</td>
<td>0.0654</td>
<td>26.0579</td>
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<tr>
<td>200</td>
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<td>50.9301</td>
<td>0.0159</td>
<td>6.2836</td>
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<td>49.9327</td>
<td>0.0000</td>
<td>0.0000</td>
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</table>

Table 1: Optimal inspection corresponding to the different samples. The last line corresponds to the true parameter value \(\lambda_{h,\text{true}} = 0.09\) failures/defect/month.

The inspection periods and downtimes are given in days.

We can see that \(\lambda_h\) converges towards \(\lambda_{h,\text{true}}\) while \(\Delta D_{\text{downtime}}\) (days) and \(\Delta C_{\text{cost}}\) (€) go to zero. However, for a very small sample size, the differences and sub-optimality are quite large.

4. Precise Bayesian Approach

In a situation where the only pieces of information we have about a parameter are its lower and upper bound, many precise Bayesians think that we ought to represent our ignorance through a uniform prior, which stems from the principle of maximum entropy Jaynes (1982). Consequently, we considered a uniform prior over the values of \(\lambda_h \in [0.01; 0.1]\) failures/defect/month we shall call \(f_{1,0}(\lambda_h)\), wherein 0 stands for prior and 1 stands for the index of the probability distribution (see Section 5). The posterior distribution given a sample \(S\) is obtained through a straightforward application of Bayes’ theorem:

\[
f(\lambda_h | S) \propto \int_{\lambda_h=0.01}^{0.1} \frac{L(S|\lambda_h) f_{1,0}(\lambda_h)}{\lambda_h f(\lambda_h) f_{1,0}(\lambda_h) d\lambda_h}
\]

(12)

Given a sample \(S\), the value of \(\lambda_h\) is estimated by

\[
E(\lambda_h | S) = \int_{\lambda_h=0.01}^{0.1} \lambda_h f(\lambda_h | S)
\]

(13)
Figure 2: Uniform prior and posteriors for $\lambda_h$.

wherein $f(\lambda_h|S)$ is the prior when $S = \emptyset$. For a given inspection period $T$ and a sample $S$, the expected value of $D_1$ with respect to $\lambda_h$ is given by Eq. 14.

$$E_{\lambda_h}(D_1(T, \lambda_h)) = \int_{\lambda_h=0.01}^{0.1} D_1(T, \lambda_h) f(\lambda_h|S)d\lambda_h$$  (14)

Likewise, the expected value of $C_1$ with respect to $\lambda_h$ is given by Eq. 15.

$$E_{\lambda_h}(C_1(T, \lambda_h)) = \int_{\lambda_h=0.01}^{0.1} C_1(T, \lambda_h) f(\lambda_h|S)d\lambda_h$$  (15)

These two quantities are the posterior expected losses with respect to the downtime and to the cost, respectively. Minimising such an expected loss function with respect to $T$ is a standard decision criterion for Bayesian decision making Robert et al. (2007). The prior and posteriors can be seen in Figure 2 and the posterior estimates of $\lambda_h$ can be found in Table 2. As could be expected, we can see that the posterior and expected value of $\lambda_h$ converge towards the real value $\lambda_{h,true} = 0.09$ failures/defect/month.

Table 3: Sub-optimality of the decisions based on the prior/samples. The last line corresponds to the true parameter value $\lambda_{h,true} = 0.09$ failures/defect/month.

<table>
<thead>
<tr>
<th>$ny$</th>
<th>$\Delta Downtime_B$</th>
<th>$\Delta Downtime_F$</th>
<th>$\Delta Cost_B$</th>
<th>$\Delta Cost_F$</th>
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<td>0.0481</td>
<td>0.0605</td>
</tr>
<tr>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

5. Imprecise Bayesian Approach

From a precise Bayesian standpoint, since we only know, in the absence of data, that $\lambda_h \in [0.01; 0.1]$ failures/defect/month, we assume that all values in that interval are equally likely so that we represent our initial ignorance through a uniform prior $f_{1,0}(\lambda_h)$ on this interval.

However, we are, logically speaking, equally ignorant about the values of any variable $Z = fun(\lambda_h)$ where $fun$ is a deterministic function (that is not too extreme). We can, with equal justification, apply the principle of indifference to

$$Z \in [\min_{\lambda_h \in [0.01; 0.1]} fun(\lambda_h), \max_{\lambda_h \in [0.01; 0.1]} fun(\lambda_h)]$$

and then deduce the cumulative probability distribution (cdf) for $Z$ and thereupon the corresponding cdf for $\lambda_h$, the derivation of which leads to a prior pdf for the values of $\lambda_h$ that is uniform, i.e., indifferent, with respect to the values of $Z$.

Let us now consider the mean delay time between the appearance of a defect and the breakdown of the pump $E(H)$. Since $H \sim \text{exp}(\lambda_h)$, we have $E(H) = \frac{1}{\lambda_h} \in [10; 100]$ months. Since we are equally ignorant about $E(H) \in [10; 100]$ months, we are, from a precise Bayesian point of view, also entitled to representing our ignorance through a uniform prior over $E(H)$ whose probability density as a
function of $\lambda_h$ is given by Eq. 16.

$$f_{2,0}(\lambda_h) = \frac{1}{\lambda_h} \frac{1}{\lambda_{h,\text{min}}} - \frac{1}{\lambda_{h,\text{max}}}$$

(16)

Given a very large number of pumps having a defect which appeared at time 0, let $\text{prop}_{\text{Safe}}(t, \lambda_h) = e^{-\lambda_h t}$ be the expected proportion of pumps still working at time $t$. For a given $t$, our maintenance engineer only knows that

$$\text{prop}_{\text{Safe}}(t, \lambda_h) \in \left[ e^{-0.1t}, e^{-0.01t} \right]$$

(17)

If he or she decides to represent his/her initial ignorance through a uniform pdf with respect to $\text{prop}_{\text{Safe}}(t, \lambda_h)$, the pdf of $\lambda_h$ is given by Eq. 18.

$$f_{3,0}(\lambda_h) = \frac{t e^{-\lambda_h t}}{e^{-0.01t} - e^{-0.1t}}$$

(18)

If we are ignorant about the value of $\text{prop}_{\text{Safe}}(t, \lambda_h)$, we are logically equally ignorant about the value of $\text{prop}_{\text{Safe}}(t, \lambda_h)$ which belongs to the interval $\left[ e^{0.01t}, e^{0.1t} \right]$. If the engineer decides to express his/her initial ignorance via a uniform prior over $\text{prop}_{\text{Safe}}(t, \lambda_h)$, the corresponding pdf with respect to $\lambda_h$ is given by Eq. 19.

$$f_{4,0}(\lambda_h) = t e^{\lambda_h t}$$

(19)

From a fundamental point of view, our engineer only knows that $\lambda_h \in [0.01; 1.0]$, which is equivalent to $E(H) \in [10; 100]$, 

$$\text{prop}_{\text{Safe}}(t, \lambda_h) \in \left[ e^{-0.1t}, e^{-0.01t} \right]$$

(20)

and

$$\frac{1}{\text{prop}_{\text{Safe}}(t, \lambda_h)} \in \left[ e^{0.01t}; e^{0.1t} \right].$$

(21)

According to objective precise Bayesianism, he or she ought to represent his/her initial ignorance through a flat prior over the variable he/she is ignorant about. Unluckily, such a flat prior for, say, $E(H)$, would be non-uniform with respect to the three other Bayesian random variables and thus express a specific knowledge about their distributions even though we are supposed to only know their bounds Norton (2008). Moreover, it would be very hard and contrived to argue that we are actually ignorant about only one of these variables (such as $\lambda_h$) and that we would be irrational if we were to feel equally ignorant about the other variables. Instead, in such a situation our genuine ignorance can only be realistically expressed through a family of prior probability distributions Walley (1990, 2000).

Since it seems to be extremely hard (if not downright impossible) to build up a prior/posterior distribution conjugate to the likelihood function given by Eq. 11, our engineer decides to represent his/her initial ignorance through a flat prior over $h$ appearing at time $0$, let

$$p_{\text{Safe}}(t, \lambda_h) = \frac{1}{\lambda_{h,\text{max}}} e^{-\lambda_h t}$$

(22)

and

$$p_{\text{Safe}}(t, \lambda_h) = \frac{1}{\lambda_{h,\text{max}}} e^{-\lambda_h t}$$

(23)

The posteriors can be seen in Figure 4-5. The lower and upper predictions for $E_t(\lambda_h|S)$ (i being the index of the prior) are compared with the frequentist and precise Bayesian estimate of $\lambda_h$ in Table 4. It can be seen that the interval
 decisions are shown in Table 5 and 6. For large sample sizes (such as $n_Y = 1000$ and $n_Y = 600$), the intervals for $T_{D, opt, S}$ and $T_{C, opt, S}$ are very narrow and the suboptimality of the frequentist and precise Bayesian decision (represented by $\Delta Cost_F$, $\Delta Downtime_F$, $\Delta Cost_B$, and $\Delta Cost_B$) is quite small.

Nevertheless, in the absence of any data, the optimal inspection period for minimising $D_1$ belongs to the interval $[51.7014 ; +\infty]$ days. This means that the optimal decision is extremely uncertain, which corresponds to the fact that the frequentist decision would lead to a sub-optimality of $\Delta Downtime_B = 9.8058$ days of downtime in comparison to the true optimal inspection period that could be calculated if we knew the real value of $\lambda_h$. Likewise, for $n_Y = 2$ failure times, the optimal inspection periods for minimising $C_1$ belongs to the interval $[54.9475 ; +\infty]$ days. This extremely strong imprecision corresponds to the fact that the frequentist and precise Bayesian decisions would be highly sub-optimal in that $\Delta Cost_B = 2568.2310 \varepsilon$ and $\Delta Cost_F = 1428.5680 \varepsilon$.

6. Discussion and Conclusions

In this work, we have developed and applied an imprecise Bayesian approach to delay time modelling for the choice of optimal inspection periods in industrial maintenance. Eight samples of virtual data have been generated and the frequentist and precise Bayesian approaches were compared with the newly developed imprecise Bayesian approach. While the three approaches give very similar results for large sample sizes ("the priors wash out" Hawthorne (1994)), they strongly differ for small sample sizes or in the absence of any data. In contrast to other imprecise decision criteria such as $\Gamma$-maximax and $\Gamma$-maximin Troffaes (2007) that would have returned a single decision value for the inspection period, E-admissibility returns a set of inspection periods. Far from being a defect, it allows the engineer to realise that when the interval returned by the imprecise probabilistic approach is very large (or even infinite), the evidential basis at his/her disposal is so thin that it is not possible to tell which inspection period would be optimal. A practical
consequence of this result is that the enterprise needs to collect more data in order to make an informed decision.

Opponents of imprecise probabilities often argue that sharp, single-valued probabilities are all we need to represent all forms of uncertainty. For instance, Lindley wrote that

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Whatever way uncertainty is approached, probability is the only sound way to think about it.

Lindley (2013). The results obtained through this study undermine that notion. The precise Bayesian method always returns a single optimal inspection period without any regard for its reliability and the level of information it is based upon. To be sure, a competent Bayesian statistician would be able to see that the evidential basis is much stronger for \( n_y = 1000 \) than for \( n_y = 2 \) because the posterior pdf of \( \lambda_b \) is much flatter in the latter case. However, the shape of the probability distribution of \( \lambda_b \) has no bearing upon the precise Bayesian optimal decision that is always based on minimising the posterior expected loss without considering the variance of the loss Bradley (2019). What is more, experience shows that time and time again many classical Bayesians draw very strong conclusions that are entirely grounded on the prior probability distribution and not on any empirical data Benetreau-Dupin (2015); Norton (2010).

A precise Bayesian who wants to avoid these pitfalls must resort to ad-hoc reasoning, which contradicts the aspiration of precise Bayesianism to be a universal holistic framework for dealing with every sort of uncertainty as expressed by Lindley (2013).

In contrast, an imprecise Bayesian approach relying on E-admissibility allows one to make a distinction between different degrees of knowledge and ignorance, as Sturgeon elegantly expressed it:

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Evidence and attitude aptly based on it must match in character. When evidence is essentially sharp, it warrants sharp or exact attitude; when evidence is essentially fuzzy -as it is most of the time- it warrants at best a fuzzy attitude.

Sturgeon (2008) It is worth noting that the present study also undermines the use of frequentist methods that are often defended on the grounds that they do not rely on a prior that is not capable of expressing genuine ignorance. Like the precise Bayesian approach, the frequentist approach also returns a single number that does not indicate how strongly the empirical evidence truly supports the decision.

Finally, while we considered a finite set of priors in this study as an easy way to demonstrate how the inclusion of imprecision can reveal the difference between ignorance, knowledge and degrees of ignorance, it might be more epistemically realistic to consider a continuous family of priors that are to be updated. This could, for example, be realised by considering the function sets

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and defining the prior set

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with \( \mathfrak{I}_0 \) becoming the vacuous prior set as \( t_{max} \rightarrow +\infty \).

Developing this thought would, however, demand much further work.

Finally, it is worth noting that the imprecise Bayesian approach developed and tested throughout this study can and should also be applied to more complex delay time models such as those involving multi-component systems where several defects can accumulate before a breakdown and where the probability distributions of the defect appearance time and the delay time are best approximated through two Weibull distributions Wang (2012a).

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References


