Fair Survival Time Prediction via Mutual Information Minimization

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Abstract
Survival analysis is a general framework for predicting the time until a specific event occurs, often in the presence of censoring. Although this framework is widely used in practice, few studies to date have considered fairness for time-to-event outcomes, despite recent significant advances in the algorithmic fairness literature more broadly. In this paper, we propose a framework to achieve demographic parity in survival analysis models by minimizing the mutual information between predicted time-to-event and sensitive attributes. We show that our approach effectively minimizes mutual information to encourage statistical independence of time-to-event predictions and sensitive attributes. Furthermore, we propose four types of disparity assessment metrics based on common survival analysis metrics. Through experiments on multiple benchmark datasets, we demonstrate that by minimizing the dependence between the prediction and the sensitive attributes, our method can systematically improve the fairness of survival predictions and is robust to censoring.

1. Introduction
Though machine learning is increasingly being used to support and perform crucial decision-making tasks, recent research has clearly demonstrated that data-driven predictive models can often retain systematic biases that are present in the underlying data and can propagate
these inequalities to their predictions. To address these issues, there has recently been a significant body of work in the machine learning community on algorithmic fairness in the context of predictive modeling, with the majority of this work focuses on classification and regression problems.

However, beyond classification and regression, there are problems in a broad range of areas, such as survival analysis, where the primary goal is to predict the time to an event of interest. Standard regression or classification models are typically inappropriate in such contexts due to censoring, where we have incomplete information about an individual’s survival time when constructing a survival analysis model. This is particularly common in medical applications for example, where individuals are followed up for different lengths of time and some individuals have not had the event of interest occur at the end of the follow-up time.

Given the broad application of survival analysis in medical applications, we briefly discuss below a number of specific motivating examples (and related issues) from the medical field. Common medical applications of survival analysis include analyzing time to death or disease recurrence in clinical trials or predicting time to re-hospitalization using Electronic Health Records (EHRs). Unbalanced representations of subpopulation groups have been frequently reported in both contexts. For example, clinical trials are often biased and are not representative of racial/ethnic minority groups (Gianfrancesco et al., 2018). In addition, most EHRs of academic hospitals, minorities, and individuals with public insurance are under-represented with smaller sample sizes, shorter follow-up time, less encounters, and fewer lab measurements (Seyyed-Kalantari et al., 2020; Chen et al., 2021). There is an emerging recognition that such biases in data often lead to the unfair performance of predictive models (Paulus and Kent, 2020; Mhasawade et al., 2021; Gervasi et al., 2022) for life-saving decisions. As another example, when model-based predictions are used to prioritize patients for rationed services (e.g., organ transplantation, specialist referrals, or intensive care unit (ICU) services), prediction disparity can lead to systematically unfair treatments for the under-represented patients (Paulus and Kent, 2020). Most of these decision-making models rank candidates by their predicted survival time, such as ranking algorithms for organ transplantation (Nilsson et al., 2015) and ICU admission triage (Iwase et al., 2022). Therefore, the problem of ensuring fairness in healthcare-related predictive models is of vital importance, as these models are already being utilized to make highly sensitive and life-changing decisions.

However, there is a critical gap between the practical use of survival analysis and the development of fairness-aware methodologies in the research literature to achieve fair survival time prediction, because survival models often output the hazard function rather than the predicted survival time. Additionally, there has been limited discussion on effective fairness measurements for survival time predictions. This paper addresses these gaps by:

- Developing a general framework for Fair Survival Time Prediction (FAST) that directly achieves Demographic Parity (DP), between the predicted survival time $\mathcal{T}_\theta$ and sensitive attributes, rather than DP on model outputs, $f(X)$. This is critical in survival analysis because $\mathcal{T}_\theta$ takes inputs from $f(X)$ and others, including baseline survival probabilities affected by censoring $C$;
Figure 1: Comparison of observed (Kaplan-Meier) and predicted survival curves for the FLChain and Framingham datasets by a non-fairness-aware model, separated by sensitive attributes. The predicted survival curves were generated using the DeepCox model, and both DNNSurv and DeepHit models showed similar patterns (see Appendix A). For both datasets, the non-fairness-aware model consistently overestimated the event risk for females, highlighting the presence of systematic bias and disparities in survival analysis. These results suggest that non-fairness-aware models may systematically produce unfair and biased outcomes across sensitive attributes in survival analysis.

- Proposing a series of comprehensive and interpretable fairness assessment metrics for survival time predictions, including concordance and calibration;
- Illustrating via empirical studies on multiple survival analysis datasets\(^1\) that the proposed FAST method can systematically improve prediction parities while maintaining reasonable degrees of overall performances;
- Demonstrating two unique advantages of FAST methodology: (1) compatibility with a broad range of existing survival analysis models, and (2) robustness when the disparity originates from censoring, which is a specific (and common) issue in survival analysis.

Motivating Example: Presence of Prediction Disparity

To assess the presence of prediction disparities, we implemented a non-fairness-aware model, the Deep Cox proportional hazard model (DeepCox), on the FLChain and Framingham datasets, where the sensitive attribute considered was “sex” (female and male). Sex was not included as a covariate in the prediction model. We compared the observed survival curves

\(^1\) Code available at https://github.com/nyumed-judy-lab/fair-survival
obtained from the Kaplan-Meier method with the predicted survival curves obtained from DeepCox for each group in each dataset.

For FLChain, the prediction for the male group closely followed its observed curve, while the predicted curve for the female group was below the observed curve (Figure 1-(a)). These findings indicate that DeepCox systematically overestimates the event risk for females but performs relatively well for males. Similar results were observed for Framingham (Figure 1-(b)). We also tried other non-fairness-aware models, including DNNSurv and DeepHit models, and they both showed similar patterns (see Appendix A). Overall, these results highlight the presence of prediction disparities, which can lead to systematic bias and unfairness in the prediction outcomes for sensitive attributes in survival analysis.

**Generalizable Insights about Machine Learning in the Context of Healthcare**

Our experience in assessing prediction disparities of survival analysis models and developing fair survival analysis models based on deep neural networks has led to a number of generalizable insights we discuss below. The primary insights are:

- In healthcare, it is important for evaluation metrics of machine learning methods to reflect their practical use if they were to be deployed. While it is common for researchers to use metrics from previous studies, it is also crucial to examine whether these metrics are realistic and clinically relevant.

- When assessing prediction disparity in a survival time prediction model, it is essential to consider various factors. For example, different forms of systematic prediction disparity, including concordance, calibration, and Brier score, may exist in a survival analysis model, as can be seen in our motivating example. It is important to note that the presence of prediction disparity in one type may not necessarily indicate its presence in other types.

- Researchers tend to overlook the importance of the baseline hazard function in the Cox proportional hazard model because of the proportional hazard assumption. However, when evaluating prediction disparity in survival analysis, it is crucial to take the baseline hazard function into account for calibration performance, such as expected $\ell_1$ calibration error, and the Brier score.

**2. Related Work**

Broadly speaking, there are three main strategies that are pursued in the algorithmic fairness research literature: (i) data pre-processing (Calmon et al., 2017; Li and Liu, 2022), (ii) in-process approaches which enforce fairness during model training (Zafar et al., 2017; Donini et al., 2018; Agarwal et al., 2019; Kleindessner et al., 2022; Shah et al., 2022; Do et al., 2022), and (iii) post-processing to adjust a model’s predictions to achieve fairness after the model training (Hardt et al., 2016; Wei et al., 2020; Chzhen et al., 2020; Soen et al., 2022). Our work fits in the in-processing approach. Early examples of this approach include fair representation learning (FRL), e.g., the work of Zemel et al. (2013) reduces DP for binary classification, with significant follow-up work for a variety of other fairness criteria and
various downstream tasks (Madras et al., 2018; Roy and Boddeti, 2019; Gupta et al., 2021; Kim et al., 2022; Shui et al., 2022).

Nonetheless, although there has been considerable prior work on FRL and other approaches for fair prediction in contexts such as classification and regression, there has been relatively little work focusing on fair survival analysis specifically. In particular, because the output of a survival analysis model is typically a hazard function (namely the instantaneous rate of risk of an event at some time t), common definitions of fairness using DP are not straightforwardly applicable because they cannot be defined directly on the output of a survival model. A number of recent papers have attempted to address this issue in different ways. Zhang and Weiss (2021, 2022) proposed the fair survival random forest model (FSRF), using a fair splitting criterion based on the log-rank test statistic. However, their approach is limited to tree-based models and requires discretization of real-valued sensitive attributes. Keya et al. (2021) developed the fair Cox proportional hazard models and associated fairness metrics by equalizing the proportional hazards and ignoring the baseline hazards, but requiring restrictive parametric assumptions of the Cox proportional hazard model. Rahman and Purushotham (2022) proposed pseudo value-based survival models by adding a fairness penalty term defined on predicted survival probability at time t. A significant limitation of this approach is that it only applies to models with a pseudo value-based loss function. Finally, Hu and Chen (2022) applied distributionally robust optimization (DRO), originally developed for binary classification (Hashimoto et al., 2018), to Cox proportional hazard model to achieve fairness. Besides, Curth et al. (2021) studied learning heterogeneous treatment effects for survival analysis, which is closely related to fairness problems.

In summary, general techniques to impose fairness in classification and regression tasks are not directly applicable to survival analysis. In addition, the (limited) prior work on fair survival analysis requires strong model assumptions and/or the use of ad-hoc fairness assessment metrics that lack clear interpretation. Our framework overcomes these limitations by using interpretable assessment metrics and being flexible and general so it can be compatible with any survival model that has a differentiable likelihood. Our approach also bridges the gap between fairness definitions and the predicted survival time; we demonstrate the utility of this via a novel set of comprehensive evaluation metrics derived from widely-used metrics for survival analysis.

3. Problem Formulation

Survival analysis involves the estimation of the probability distribution of time-to-event in the presence of censoring. In a typical survival analysis setting, for each individual, we have realizations of covariates $X$, a follow-up time $O$, and an event indicator $\Delta$, which takes value 1 if the event precedes the censoring and 0 otherwise. In addition, we have $K$ groups corresponding to $K$ possible values in $A = \{\alpha_1, \cdots, \alpha_K\}$ of a sensitive attribute $A$ such as race/ethnicity or gender $A$. Thus, we have a dataset $D = \{(x_i, a_i, o_i, \delta_i) \in X \times A \times \mathbb{R}_+ \times B : i = 1, \cdots, n\}$ where $x_i$, $a_i$, $o_i$, and $\delta_i$ are realizations of $X$, $A$, $O$, and $\Delta$ for individual $i$.

In this paper, we adopt the usual independent censoring assumption in the survival analysis. Let $T$ and $C$ be the event and censoring time, respectively, so that $O = \min\{T, C\}$ and $\Delta = I(T \leq C)$. 

5
Under the independent censoring assumption, we learn a model, parameterized by $\theta$, by minimizing the negative log-likelihood: 

$$\hat{\theta}_{\text{MLE}} = \arg\min_{\theta} -\ell(\theta; D),$$

where

$$\ell(\theta; D) = \frac{1}{n} \sum_{i=1}^{n} \left[ \delta_i \log \lambda_\theta(o_i|x_i) - \Lambda_\theta(o_i|x_i) \right],$$

and $\Lambda_\theta(\cdot|x)$ is the cumulative hazard function such that $\Lambda_\theta(t|x) = -\log S_\theta(t|x)$, for all $t \geq 0$. $S_\theta(t|x) = P(T > t|x)$ is the survival function given $x$, and $\lambda_\theta(t|x)$ is the conditional hazard function defined as $\frac{d}{dt} \Lambda_\theta(t|x)$. The actual form of the hazard function depends on the choice of model.

This conventional MLE approach for survival analysis often results in dependence between the prediction and the sensitive attributes, which may lead to disparity of prediction performance among the sensitive groups. To mitigate the prediction disparity, we can add a penalty term $P(\theta; D)$, yielding the following penalized negative log-likelihood problem:

$$\hat{\theta}_{\text{FAST}} = \arg\min_{\theta} -\ell(\theta; D) + \gamma P(\theta; D),$$

where $\gamma > 0$ is a hyperparameter. By jointly minimizing the negative log-likelihood for prediction as well as the fairness penalty term, we can obtain a survival model that has less prediction disparity compared to the conventional MLE approach. In the following section, we introduce how to formulate the fairness penalty term.

### 3.1. DP for Survival Analysis

One difference between the survival analysis and other prediction tasks is that a survival model produces the conditional hazard value conditioned on $x$, while models for other tasks typically output the conditional expected outcome value given $x$. We denote the predicted survival time as a random variable $T_\theta$, distributed over $\mathbb{R}^+$, from a survival analysis model parameterized by $\theta$. The cumulative probability distribution function of $T_\theta$ is defined by the cumulative hazard function as $P(T_\theta(x) \leq t) = 1 - \exp(-\Lambda_\theta(t|x))$.

**Definition 1 (Demographic Parity (Kamiran and Calders, 2009))** We say the survival analysis model, parameterized by $\theta$, satisfies demographic parity if $T_\theta$ is statistically independent of the sensitive attribute $A$, i.e., $A \perp \perp T_\theta$. In other words, $S_\theta(t|A = \alpha_j) = S_\theta(t|A = \alpha_k)$ for any $t \in [0, \infty)$ and $\alpha_j, \alpha_k \in A$.

To achieve DP, we need to minimize the dependence between the sensitive attribute $A$ and the predicted survival time $T_\theta$. The dependence between two random variables can be quantified in several ways. For example, mutual information (MI), which quantifies the amount of information obtained about one random variable by observing the other one, captures non-linear statistical dependencies between random variables and is often considered as a measure of true statistical dependence (Kinney and Atwal, 2014). Some alternative choices for the dependence measure are discussed in Appendix B.1.

**Definition 2 (Mutual Information of $A$ and $T_\theta$)** Mutual information that quantifies the dependence of the sensitive attribute $A$ and the predicted survival time $T_\theta$ is defined as

$$\mathcal{I}(A, T_\theta) = \int_{A \times \mathbb{R}} \log \frac{dP_{A T_\theta}}{dP_A \otimes dP_{T_\theta}} dP_{A T_\theta},$$

where

$$\ell(\theta; D) = \frac{1}{n} \sum_{i=1}^{n} \left[ \delta_i \log \lambda_\theta(o_i|x_i) - \Lambda_\theta(o_i|x_i) \right],$$
where \( P_{AT_0} \) is the joint CDF, \( P_A \) and \( P_{T_0} \) are the marginal CDFs of \( A \) and \( T_0 \) respectively. MI is nonnegative, and a smaller value implies a weaker dependence between \( A \) and \( T_0 \). Moreover, \( I(A, T_0) = 0 \) if and only if \( A \perp \perp T_0 \).

We propose a **FAir Survival Time** (FAST) prediction framework, as the minimizer of the weighted sum of the negative log-likelihood and the MI between \( A \) and \( T_0 \):

\[
\hat{\theta}_{\text{FAST}} = \arg\min_{\theta} - \ell(\theta; D) + \gamma I(A, T_0). \tag{4}
\]

We note that formulating the fair learning problem as MI minimization is prevalent in the literature (Creager et al., 2019; Song et al., 2019; Zhu et al., 2021; Grari et al., 2021; Zheng and Li, 2022), however, none of them can be applied to the survival analysis directly.

### 3.2. Mutual Information Estimation

Since the mutual information of \( A \) and \( T_0 \) will often not have a closed form, we adopt a non-parametric approach, in particular, the Mutual Information Neural Estimation (MINE) proposed by Belghazi et al. (2018). This allows us to estimate the mutual information from data samples without assuming a distributional form of \( A \) or \( T_0 \).

The main idea of MINE is to maximize the lower bound of the mutual information utilizing its dual representation (Belghazi et al., 2018). Let \( \{a_i \in A : i = 1, \ldots, n\} \) be a set of sensitive attributes and \( \{\tau_i \sim T_0(x_i) : i = 1, \ldots, n\} \) be a set of samples drawn from \( T_0(x_i) \), respectively. Let \( \{a'_i \in A : i = 1, \ldots, n\} \) be another set of sensitive attributes obtained by randomly rearranging the elements of \( \{a_i \in A : i = 1, \ldots, n\} \). The estimate of mutual information \( \hat{I}(A, T_0; D) \) of \( A \) and \( T_0 \) can be obtained through maximizing its lower bound \( \hat{I}_\omega(A, T_0; D) \)

\[
\hat{I}(A, T_0; D) = \sup_\omega \hat{I}_\omega(A, T_0; D) = \sup_\omega \frac{1}{n} \sum_{i=1}^{n} \psi_\omega(a_i, \tau_i) - \log \left[ \frac{1}{n} \sum_{i=1}^{n} \exp \psi_\omega(a'_i, \tau_i) \right], \tag{5}
\]

where \( \psi_\omega : (a_i, \tau_i) \mapsto \mathbb{R} \) is a function parameterized by a neural network with parameters \( \omega \). Note \( (a_i, \tau_i) \) is a sample drawn from the joint distribution \( P_{AT_0} \) and \( (a'_i, \tau_i) \) is drawn from the product of the marginals \( P_A \otimes P_{T_0} \). Belghazi et al. (2018) proved that MINE is a strongly consistent estimator of mutual information and linearly scalable in both dimensionality and sample size. Moreover, it is trainable through backpropagation, so that we can plug it into Equation (7).

### 3.3. Log-likelihood

FAST does not require a specific form of log-likelihood and is compatible with any survival analysis model with a differentiable log-likelihood. If one chooses to use the Cox proportional hazard (PH) model, the log-likelihood is

\[
\ell(\theta; D) = \frac{1}{n} \sum_{i=1}^{n} \left[ \delta_i (\log \lambda_0(a_i) + f_\theta(x_i)) - \Lambda_0(a_i) \exp f_\theta(x_i) \right], \tag{6}
\]

where \( \lambda_0 \) and \( \Lambda_0 \) are the baseline hazard function and the cumulative baseline hazard functions respectively, and \( f_\theta : \mathcal{X} \rightarrow \mathbb{R} \) is the log proportional hazard function which is parameterized by parameters \( \theta \). We present several alternative choices for the log-likelihood function in Appendix C.
Algorithm 1 SGD for FAST

1: **Input**: Data $\mathcal{D} = \{(x_i, a_i, o_i, \delta_i)\}_{i=1}^{n}$, hyperparameter $\gamma$, learning rates $\eta_\theta, \eta_\omega$, batch size $b$.
2: **Output**: $\hat{\theta}_{\text{FAST}}$ solving (7).
3: $\theta^{(0)}, \omega^{(0)} \leftarrow$ initialize
4: while not converged do
5: Draw a minibatch of samples from $\mathcal{D}$: $\mathfrak{B} = \{(x_i, a_i, o_i, \delta_i)\}_{i=1}^{b}$
6: Draw time-to-event from $T_{\theta^{(t)}}(x_i)$: $\{\tau_i\}_{i=1}^{b}$
7: Draw another minibatch of samples from $A$: $\{a'_i\}_{i=1}^{b}$
8: Prepare a minibatch for $\tilde{I}_\omega$: $\mathfrak{C} = \{(a_i, \tau_i, a'_i)\}_{i=1}^{b}$
9: Update $\theta$ by descending its stochastic gradient.
   $\theta^{(t+1)} \leftarrow \theta^{(t)} - \eta_\theta \nabla \ell(\theta^{(t)}; \mathfrak{B}) - \gamma \nabla \tilde{I}_\omega(A, T_{\theta^{(t)}}; \mathfrak{C})$
10: Update $\omega$ by ascending its stochastic gradient.
    $\omega^{(t+1)} \leftarrow \omega^{(t)} + \eta_\omega \nabla \tilde{I}_\omega(A, T_{\theta^{(t)}}; \mathfrak{C})$
11: end while
12: $\hat{\theta}_{\text{FAST}} = \theta^{(t)}$

3.4. Estimation of FAST

With Equation (5), the estimation of FAST becomes:

$$\min_\theta \max_\omega - \ell(\theta; \mathcal{D}) + \gamma \tilde{I}_\omega(A, T_\theta; \mathcal{D}).$$

We solve the minimax problem using minibatch stochastic gradient descent (SGD) optimization. Given each batch, we simultaneously update $\theta$ and $\omega$ by ascending and descending their gradients, as illustrated in Algorithm 1. At each iteration, the algorithm randomly draws minibatches of size $b$ from $P_{XAOA}$ and $P_A$. Then, it generates random samples from $T_\theta(x_i)$ for each $i$. The exact method to sample $\tau_i$ differs by choice of model, but in most cases, we can draw samples using inverse transform sampling (see Appendix C for details). Then, we take random samples drawn from the joint distribution $P_A \otimes P_{T_\theta}$ and the product of marginal distributions $P_A \otimes P_{T_\theta}$ and plug them into (5). This allows us to calculate the gradient of $\tilde{I}_\omega(A, T_\theta; \mathcal{D})$ with respect to $\omega$ and $\theta$, respectively. As studied in Belghazi et al. (2018), the naive minibatch SGD gradients for $\tilde{I}_\omega(A, T_\theta; \mathcal{D})$ are biased, so we adopt the moving average technique to reduce the bias.

The gradient of $\ell$ with respect to $\theta$ can be straightforwardly computed. For the CoxPH log-likelihood function (6) with $f_\theta$ parameterized by a neural network, we first estimate the baseline hazard functions $\hat{\lambda}_0$ and $\hat{\Lambda}_0$. We then take the full log-likelihood approach to solve $\ell(\theta; \mathcal{D}, \hat{\lambda}_0, \hat{\Lambda}_0)$.

3.5. Choice of the Hyperparameters

We take a general strategy to select the hyperparameter, namely to minimize the disparity while placing constraints on the overall predictive performance. Specifically, we evaluate the performances and disparities of models trained with different $\gamma$ values on a held-out validation set, and choose the $\gamma$ with the smallest disparity while maintaining the overall performance within a pre-determined acceptable margin, such as $\pm 5\%$, of that under the vanilla model.
with $\gamma = 0$. In order to investigate the sensitivity of metrics for hyperparameter-tuning purposes, we propose a series of evaluation metrics in Section 4, and compare their empirical performances in Section 5. We note that Keya et al. (2021) considered a similar strategy for selecting the fairness hyperparameter.

4. Fairness Assessment for Survival Analysis

The prediction performance of a survival model is usually assessed by concordance, discrimination, and calibration. There has been limited discussion to date in the survival analysis literature on what metrics are recommended to assess the disparity of prediction performances among sensitive attribute groups. One straightforward assessment is to calculate the average (absolute) differences of any metric between the sensitive attribute groups. In this section, we discuss that this is often insufficient, especially for concordance and discrimination metrics.

We will denote the results from a survival analysis model as $\{(o_i, r_i, \delta_i) : i = 1, \ldots, n\}$, where $o_i = \min\{c_i, t_i\}$ is the observed follow-up time, $r_i = P(T_\theta(x_i) < t)$ is the predicted risk at time $t$ provided by a survival analysis model parameterized by $\hat{\theta}$, and $\delta_i = I(t_i \leq c_i)$ is the observed event indicator.

4.1. Concordance

The concordance index (C-index) is a widely used metric for evaluating survival analysis models (Harrell et al., 1982, 1984, 1996). It captures the rank correlation between the survival time and predicted risks – a survival model tends to assign high-risk individuals who have shorter survival times with greater C-index. C-index is calculated as the proportion
of the number of concordant pairs out of the number of valid pairs $C_i = \frac{|S_c|}{|S_v|}$, where $S^v = \{(i, j) : o_i \leq o_j, \delta_i = 1\}$ is the set of valid pairs, and $S^c = \{(i, j) \in S^v : r_i > r_j\}$ is the set of concordant pairs.

However, to evaluate prediction concordance for subjects in a subgroup, there are two versions of the C-index, differing in how the valid and concordant pairs are defined. If one considers only the subjects in a sensitive attribute group $k$, the sets of valid pairs and concordant pairs are defined as $S^v_k = \{(i, j) : o_i \leq o_j, \delta_i = 1, a_i = a_j = \alpha_k\}$ and $S^c_k = \{(i, j) \in S^v_k : r_i > r_j\}$ respectively. We refer to this as the stratified group C-index, $C_{ik} = \frac{|S^c_k|}{|S^v_k|}$.

The stratified group C-index measures how a model correctly arranges the risks and times-to-event within the group. However, it cannot capture the disparity of group $k$ relative to other groups. For instance, considering a model that underestimates one group’s risk compared to another (Figure 2-(a)), its stratified C-indices for all groups can be good, even if the rank correlation between the observed survival time and predicted risks are relatively worse for the group.

This encourages us to consider another group-level C-index to reflect the macro-level assessment. We define the set of valid pairs and concordant pairs by $S^v_k = \{(i, j) : o_i \leq o_j, \delta_i = 1, a_i = \alpha_k\}$ and $S^c_k = \{(i, j) \in S^v_k : r_i > r_j\}$. Note that this definition pairs instances of a specific group with pooled candidates from the entire dataset. Thus, we refer to this as the pooled group C-index, $C_{ik}^p = \frac{|S^c_k|}{|S^v_k|}$.

In Figure 2-(a), $C_{i2}^p$ is significantly worse than that of Group 1. However, in another situation when a model predicts like Figure 2-(b), $C_{i2}^p$ cannot capture the within-group concordance, while the stratified group C-index can. Therefore, we argue that disparities of both versions of C-indices are needed to comprehensively measure the concordance disparity. We define the stratified group C-index disparity $\Delta C_{it}$, as the maximum of pairwise absolute differences of stratified group C-indices from all possible groups, and similarly for the pooled group C-index disparity $\Delta C_{ip}$. Note that, $\Delta C_{ip}$ has also been considered in Zhang and Weiss (2021) as the “concordance imparity.” Also, a similar comparison between the stratified and pooled group AUCs, in the context of binary classification, was discussed in Yang et al. (2023).

### 4.2. Calibration

Calibration is an important component for assessing survival time prediction models, which measures the agreement between predicted probabilities and observed event rates within a given duration of time. Different methods have been proposed to evaluate the calibration of survival models. They typically involve constructing a calibration curve by dividing individuals into subsets based on predicted event probabilities and comparing the mean predicted event probabilities with the observed event prevalence for each subset. Graphical representations are often used to compare the results with a diagonal line that represents perfect calibration. For more detailed information on constructing and summarizing calibration curves for survival analysis, refer to Austin et al. (2020); Haider et al. (2020); Goldstein et al. (2020) and references therein. In our study, we quantify the calibration performance of survival models using the expected $\ell_1$ calibration error (ECE). The ECE is a popular metric for classification problems using deep neural networks (Guo et al., 2017), and it is
Table 1: Benchmark datasets and their statistics. $p$ is the number of covariates, $n$ is the number of instances, $A$ is the sensitive attribute, and $C$ is the percentage of censoring.

<table>
<thead>
<tr>
<th>Datasets</th>
<th>Outcome</th>
<th>$p$</th>
<th>$A$</th>
<th>$n(C)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLChain</td>
<td>Time to Death</td>
<td>17</td>
<td>Female</td>
<td>4,347 (73.3)</td>
</tr>
<tr>
<td>(Kyle et al., 2006)</td>
<td></td>
<td></td>
<td>Male</td>
<td>3,524 (72.5)</td>
</tr>
<tr>
<td>Framingham</td>
<td>Time to Death 5</td>
<td>Female</td>
<td>2,650 (75.5)</td>
<td></td>
</tr>
<tr>
<td>(Mahmood et al., 2014)</td>
<td></td>
<td></td>
<td>Male</td>
<td>2,049 (59.8)</td>
</tr>
<tr>
<td>SUPPORT</td>
<td>Time to Death 66</td>
<td>White</td>
<td>7,191 (31.2)</td>
<td></td>
</tr>
<tr>
<td>(Knaus et al., 1995)</td>
<td></td>
<td>Black</td>
<td>1,391 (34.7)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hispanic</td>
<td>290 (40.3)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Other</td>
<td>191 (27.2)</td>
<td></td>
</tr>
</tbody>
</table>

also used to evaluate the calibration performance of deep survival models (Nagpal et al., 2021). The ECE calculates the average absolute difference between the mean predicted event probabilities and observed event rates conditional on the predicted event probability. The ECE at time $t$ is defined as:

$$\text{ECE} = \frac{1}{n} \sum_{j=1}^{q} \frac{|Q_j|}{n} \left| (1 - \text{KM}_j(t)) - \bar{r}_j \right|, \quad (8)$$

Here, we divide the predicted risk scores into $q$ quantiles (bins) $Q_j = [r_j, r_{j+1})$ for $j = 1, \cdots, q$, estimate the Kaplan-Meier survival probabilities $\text{KM}_j$, and calculate the average risk score $\bar{r}_j = \frac{1}{|Q_j|} \sum_{i \in Q_j} r_i$ for each bin $Q_j$. We define the disparity of the ECE, $\Delta\text{ECE}$, as the maximum pairwise absolute difference of ECEs across all possible groups. Note that Zhang and Weiss (2022) considered hypothesis testing for fair calibration as a series of Hosmer-Lemeshow goodness-of-fit tests for each group.

4.3. Brier Score

The Brier score (Byers et al., 1951) is the most widely used metric to evaluate survival models’ concordance and calibration performances (Murphy, 1972; DeGroot and Fienberg, 1983; Haider et al., 2020). It is defined as the average squared distance between the observed survival status and the predicted survival probability.

$$\text{BRIER} = \frac{1}{n} \sum_{i=1}^{n} [(1 - r_i)^2 \mathbb{I}(o_i \leq t) \delta_i + (0 - r_i)^2 \mathbb{I}(o_i > t)].$$

Unlike the C-index, the Brier score does not depend on the relative rank of the subjects. Therefore, we can define the disparity of the Brier score, $\Delta\text{BRIER}$, as the maximum of the pairwise absolute difference of Brier scores across all possible pairs of groups.
5. Experiments

We performed comprehensive experiments to evaluate FAST on the four proposed disparity metrics using three real-world medical datasets: FLChain, Framingham, and SUPPORT. We followed the SurvSet repository (Drysdale, 2022) protocols to preprocess the datasets. Table 1 shows a summary of these datasets; details are in Appendix D.1.

We implemented FAST on three baselines (fairness-unaware) neural survival models (DeepCox, DNNSurv, and DeepHit), all of which have been shown to achieve competitive overall predictive performance in the survival analysis literature. We also compared our method with existing fair survival analysis models: GFDeepCox and IFDeepCox (Keya et al., 2021), GFDNNSurv, and IFDNNSurv (Rahman and Purushotham, 2022), and DRODeepCox (Hu and Chen, 2022). Details for all of the implementations can be found in Appendix D.2 and D.3.

5.1. Comparative Experiments

We split each dataset into three mutually exclusive sets, 60% for training, 20% for validation, and 20% as testing sets. We used the training set to estimate the models and selected hyperparameters using the validation set. We then evaluated the models’ performances and the disparities in the testing set. We calculated each metric for time points ranging from the 1st to the 99th percentile and then reported the average of those values. Detailed information, including neural network structures and optimization settings, is in Appendix D.3.

We implemented our hyperparameter selection strategy, which we introduced in Section 3.5, by setting an acceptance margin of $\pm 5\%$. In particular, we selected the hyperparameter that achieves the smallest $\Delta$Brier while ensuring that overall Brier scores do not increase by more than 5%. We choose to use the Brier score as it is a commonly used metric that measures both calibration and concordance (Haider et al., 2020).

In Table 2, all group fairness-encouraging methods, except for individual fairness models, effectively decreased the group fairness metric $G_f$ (defined as the maximum difference of expected per-group survival functions) as expected. We present trade-off curves between the $G_f$ and performance metrics in Figures A5, A6, and A7 to show the effectiveness of each method in decreasing $G_f$. Moreover, FAST not only improved parity measured in $G_f$ but also effectively decreased disparities in most of the metrics while maintaining the Brier score within the pre-defined acceptance margin. Similar levels of (or even better) efficacy of FAST were observed for the baselines, demonstrating that FAST can be flexibly used for all three types of baseline survival models. In contrast, GFDeepCox, IFDeepCox, and DRODeepCox achieved comparable performance in reducing disparities in FLChain but did not decrease $\Delta$Brier. Moreover, they showed substantially worse overall predictive performance in terms of the Brier score (marked orange when outside the acceptable margin). This may be due to GFDeepCox, IFDeepCox, and DRODeepCox placing emphasis on equity in proportional hazards but not taking the baseline hazards into account. GFDNNSurv and IFDNNSurv worked almost as efficiently as ours in both encouraging predictive parity while not losing their efficacy because they encourage parity of the pseudo-survival probabilities, unlike GFDeepCox, IFDeepCox, and DRODeepCox.
<table>
<thead>
<tr>
<th>Dataset</th>
<th>Model</th>
<th>Disparity↓</th>
<th>Performance</th>
<th></th>
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</thead>
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<tr>
<td></td>
<td></td>
<td>$G_f$</td>
<td>$\Delta Brier$</td>
<td>$\Delta CE$</td>
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<td></td>
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<td>$\downarrow$</td>
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<td>Framingham</td>
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<td>5.69</td>
</tr>
<tr>
<td></td>
<td>FASTDNNSurv (Ours)</td>
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<td>-24.7%</td>
<td>-25.0%</td>
</tr>
<tr>
<td></td>
<td>GFDeepSurv</td>
<td>22.8%</td>
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<td>5.6%</td>
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<td></td>
<td>IFDeepSurv</td>
<td>-0.1%</td>
<td>0.0%</td>
<td>-0.3%</td>
</tr>
<tr>
<td></td>
<td>Vanilla DeepHit</td>
<td>0.67</td>
<td>2.15</td>
<td>7.48</td>
</tr>
<tr>
<td></td>
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<td>-1.4%</td>
<td>-1.1%</td>
<td>4.3%</td>
</tr>
<tr>
<td>SUPPORT</td>
<td>Vanilla DNNSurv</td>
<td>0.55</td>
<td>3.34</td>
<td>8.33</td>
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<tr>
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<td>-24.7%</td>
<td>-5.6%</td>
<td>4.3%</td>
</tr>
</tbody>
</table>

Table 2: Experimental results in performance and disparity metrics from three benchmark datasets. We report actual performances as percentages for the plain baseline methods (Vanilla DeepCox, DNNSurv, and DeepHit). In contrast, for the fairness-aware methods, we show the relative change of each metric compared to its baseline method. Each number represents the average performance across 10 repeated runs. The $\downarrow$ symbol indicates that lower values are better, while the $\uparrow$ symbol implies that higher values are better. The best results for each baseline method in each dataset are highlighted in Green. Orange highlights (in the performance columns) mean each fair method’s performance is outside the 5% margin.
Figure 3: Improved Survival Predictions from FAST. The figure compares the observed survival curves (by Kaplan-Meier), predicted survival curves by DeepCox (blue and red dash lines), and predicted survival curves by FAST-DeepCox (purple and brown solid lines) for each sensitive attribute group in the FLChain and Framingham datasets. FAST improves the model’s predictions for females and males in FLChain, resulting in predicted survival curves that are closer to their observed counterparts. In Framingham, FAST reduces the bias in the DeepCox prediction, moving the predicted curve towards the middle of the two observed curves. The prediction improvements by FAST from other methods (DNNSurv and DeepHit) are shown in Figures A3 and A4.

For FAST, γ selected by each metric generally achieved the desired performance for that metric and other metrics in the test set. Although the best performances of a specific metric in the test data are usually achieved by the γ selected by the same metric, \( \gamma_{\text{Brier}} \) empirically achieved satisfactory performances on calibration and concordance.

5.2. FAST Robustness against Disparity in Censoring

In survival analysis, a unique but common source of disparity is the difference in proportions of censored subjects between the sensitive attribute groups. If one group has a shorter survival time and/or shorter follow-up time, it will lead to a higher proportion of censoring, which may result in worse prediction performance for that group. To study the robustness of FAST, we designed an experiment on FLChain and Framingham datasets to simulate this scenario. In FLChain, both females and males were followed up to 15 years from the study entry. We truncated the maximum follow-up years for males to \( C_{\text{max}} = 15, 13, 11, 9, \) and 7 years in the training and validation datasets. In the truncated dataset, we set a male subject \( i \) to censored status \( (o_i, \delta_i) = (C_{\text{max}}, 0) \) if \( o_i > C_{\text{max}} \). Thus, the proportion of censoring in males increased and became more disparate from females with decreasing \( C_{\text{max}} \).
Figure 4: Model performances and disparities (relative changes in percentage compared to the corresponding baselines) versus max follow-up years for males for the FLChain and Framingham datasets averaged over 5 test sets. The green shades represent the 5% acceptance margin of the Brier score.
We present the follow-up distribution for the event and censored individuals in Figure A11. We implemented the various methods on the truncated dataset and selected the optimal $\gamma_{\text{Brier}}$ by setting a 5% acceptance margin. In Framingham, both females and males were followed up to 35 years from the study entry. We truncated the maximum follow-up years for males (minority group) to $C_{\text{max}} = 35, 30, 25, 20,$ and 15 years in the training and validation datasets, as same as the previous. Follow-up time distributions are presented in Figure A12.

Figure 4 confirms our hypothesis, in both datasets, that disparate follow-up time directly results in disparate Brier scores from the fairness-unaware survival models. However, FAST methods achieve consistent and robust improvements in Brier score disparities across the range of truncated follow-up times while maintaining acceptable overall Brier scores (marked green margins). In contrast, the fairness-aware competitors are not as effective in decreasing disparities or maintaining overall Brier scores. We also noticed that the disparate follow-up time has relatively less impact on concordance metrics (Figures A13 and A15 in Appendix) but worsens the calibration (Figures A14 and A16 in Appendix). We conjecture that this is because disparate follow-up time impacts disparity in baseline survival functions and hazard ratio estimates. Therefore, fairness approaches that impose parity on the hazard parameters (such as GFDeepCox, IFDeepCox, and DRODeepCox) are less effective in this setting. This empirically demonstrates the advantage of directly achieving parity on the predicted survival time via a nonparametric MI estimation, which makes FAST robust against a range of possible disparities, including disparities in baseline survival probabilities and hazard functions.

6. Conclusion and Discussion

We developed a Fair Survival Time Prediction (FAST) method, via a mutual information penalty term to learn survival time predictions, which is compatible with survival analysis formulations with differentiable log-likelihood functions, including recent deep survival analysis models. We also proposed a series of metrics to evaluate parity in survival time predictions. FAST shows empirical consistency in improving prediction parity while maintaining overall prediction performance. Furthermore, we demonstrate that FAST is robust to the presence of disparity in censoring and follow-up times, which is common and essential for survival analysis applications.

Future Directions An interesting extension of work would be to consider other fairness notions such as Equalized Odds (Hardt et al., 2016). This will relax the assumption requiring independence between true survival time and $A$, which has the implications of considering diseases that have different impacts on different gender/races due to genetic reasons. However, such extensions are not trivial as the true survival time $T$ is not observed because of $C$.

Another potential future direction is to investigate the impact of the informative censoring (Lagakos, 1979) on the disparity in survival analysis and to develop a method to mitigate it.

Limitations As our method relies on MINE (Belghazi et al., 2018), which utilizes the deep neural network to estimate mutual information, it cannot be as simplified as penalized linear models. Even if our approach can be combined with a linear survival model, it still requires a nonlinear neural network to estimate MI. Another limitation of our framework is that we did not impose fairness on an individual level, so some individuals might get
disadvantaged by considering the fairness between groups. We encourage future work in this field to take this into account.

**Societal Impact** In areas such as healthcare, ensuring fairness in predictive models is essential given that these models are utilized to make highly sensitive, life-changing, and even life-saving decisions. Most of these models make decisions based on predicted survival time. Our methodology and proposed parity metrics provide a new perspective and framework to systematically improve fairness for survival predictions.

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**References**


Fair Survival Time Prediction via Mutual Information Minimization


Appendix A. Comparison of the Observed and Predicted Survival Curves

Figure A1: This figure compares observed survival curves (generated using Kaplan-Meier analysis) with predicted curves (generated using DNNSurv and DeepHit models) for two datasets: FLChain and Framingham. The results indicate that for FLChain, both DNNSurv and DeepHit models tend to underestimate the survival probability (or overestimate the risk of the event) for females. Similarly, for Framingham, both models tend to overestimate the survival probability for females. These findings suggest that the prediction disparity is not specific to the Cox proportional hazard model but rather a systematic issue that is inherent to the data.
Appendix B. Further Discussions

B.1. Alternative Statistical Dependence Metrics (Supplement for Section 3.1)

Here, we give a brief discussion on metrics to quantify statistical dependence other than mutual information. DP requires the predicted survival time $T_\theta$ to be statistically independent of the sensitive attributes $A$. Given the set of possible sensitive attributes $A = \{\alpha_1, \cdots, \alpha_K\}$, the independence of $T_\theta$ and $A$ translates into $T_\theta = (T_\theta | A = \alpha_k)$ for all $k = 1, \cdots, K$. Therefore, $T_\theta$ is independent of $A$ if and only if $\mathbb{D}(T_\theta, T_\theta | A = \alpha_k) = 0$ for all $k = 1, \cdots, K$, where $\mathbb{D}$ is a distance or divergence between two probability distributions, such as Kullback-Leibler divergence, Jensen-Shannon divergence, total variation distance, Wasserstein distance, and maximum mean discrepancy (Gretton et al., 2012). Therefore, we can include one of the following terms as a DP encouraging penalty:

$$\sum_{k=1}^{K} \mathbb{D}(T_\theta, T_\theta | A = \alpha_k), \quad \text{or} \quad \max_{k} \mathbb{D}(T_\theta, T_\theta | A = \alpha_k).$$ (A1)

The penalty term equals zero means $T_\theta$ and $A$ are statistically independent, the same as when mutual information is zero. Therefore, the distances and divergences introduced above can be used as an alternative to achieve DP. However, even if minimizing them toward zero implies the statistical independence of $T_\theta$ and $A$, each divergence or distance may generate a different trajectory of solutions obtained by changing the hyperparameter that controls the trade-off between the negative log-likelihood and the penalty term. We leave this as a future direction.

B.2. Comparison Against Fair Representation Learning

In the fairness literature, a line of work encourages the model output to satisfy some fairness criteria by learning a fair representation that can be transferred to fair performances for various downstream tasks. Zemel et al. (2013) proposed the first FRL method to achieve DP for binary classification. Further extensive work has been done recently for sophisticated fairness criteria other than DP, as well as for various downstream tasks (Madras et al., 2018; Roy and Boddeti, 2019; Gupta et al., 2021; Kim et al., 2022; Shui et al., 2022). The fair representation learning to achieve the DP aims to learn a function $\psi : \mathcal{X} \rightarrow \mathcal{Z}$ such that $\mathcal{Z} \perp \perp A$, and build a model $\phi : \mathcal{Z} \rightarrow \mathcal{Y}$ for the downstream task (usually prediction model) on top of $\psi$. This approach may look similar to our approach as encouraging the independence of $\mathcal{Z} = \psi(\mathcal{X})$ and $A$ transfers to encouraging the independence of $(\phi \circ \psi)(\mathcal{X}) = \hat{Y}$ and $A$.

However, in the survival analysis setting, the predicted survival time $T_\theta$ is not exactly the outcome of a model (or a network). In general, the model output is the hazard function, event probability, or survival probability. Therefore, the model outcome is independent of the sensitive attribute is totally different from the predicted survival time is independent of the sensitive attribute. Moreover, some survival models (e.g., Cox proportional hazard model or accelerated failure time model) have the baseline hazard function, which is totally separated from the model output. For such models, encouraging any kind of fairness, including independence, of the model output will not result in the fairness of the predicted survival time because it does not account for the baseline hazard.
B.3. Comparison Against Group Fair DeepCox

Group Fair DeepCox (Keya et al., 2021) encourages the expected proportional hazard to be the same across the sensitive groups, that is,

$$\mathbb{E}[\exp f_\theta(X|A = \alpha_k)] = \mathbb{E}[\exp f_\theta(X)],$$

(A2)

for all \(k = 1, \cdots, K\). We note that this criterion is different from DP, which requires the predicted survival time to be independent of the sensitive attribute. Equalizing the expectation of the group-conditional hazard functions across all groups does not necessarily result in the same distributions of group-conditional predicted survival times (but the reverse is true). Furthermore, (A2) is the equivalence in proportional hazard without considering the baseline hazard, so the criterion does not imply the expected hazard functions being the same. Thus, (A2) is a much weaker condition than DP, and it cannot account for the disparity involving the baseline hazard function.

Appendix C. FAST Formulations (Supplement for Sections 3.3 and 3.4)

We introduce complete formulations of our FAST approaches by specifying the log-likelihood functions for some representative survival analysis models.

C.1. Notations

Dataset \(\{(x_i, a_i, o_i, \delta_i) : i = 1, \cdots, n\}\), where \(x_i\) is the covariate vector, \(a_i\) is the sensitive attribute, \(o_i\) is the observed time (minimum of time to event or censoring), and \(\delta_i\) is the event indicator which takes 1 if the event precedes the censoring and 0 otherwise.

Model \(f_\theta : \mathcal{X} \rightarrow \mathbb{R}\), parameterized by \(\theta\). Either linear model or neural network.

Log-likelihood

$$\ell(\theta; \mathcal{D}) = \frac{1}{n} \sum_{i=1}^{n} \left[ \delta_i \log \lambda_\theta(o_i|x_i) - \Lambda_\theta(o_i|x_i) \right].$$

(A3)

In the case of discrete-time models (such as DeepHit), the negative log-likelihood becomes

$$\ell(\theta; \mathcal{D}) = \frac{1}{n} \sum_{i=1}^{n} \left[ \delta_i \log P(T_\theta(x_i) = o_i) - (1 - \delta_i)P(T_\theta(x_i) > o_i) \right].$$

(A4)

C.2. Cox Proportional Hazard Model (CPH)

Log-likelihood Cox proportional hazard model (Cox, 1972) is based on the proportional hazard assumption, which is \(\lambda_\theta(t|x) = \lambda_0(t)\lambda_\theta(x)\), where \(\lambda_0\) is the baseline function and \(\lambda_\theta\) is the proportional hazard function. We introduce a neural network \(f_\theta : x \mapsto \log \lambda_\theta(x)\) which outputs a log-hazard value given an input \(x\).

The most widely used loss function to learn Cox proportional hazard models, for both linear (Cox, 1972) and deep neural network (Katzman et al., 2018) models, is the negative partial log-likelihood, which is defined as follows:

$$-\ell_p(\theta; \mathcal{D}) = -\frac{1}{n} \sum_{i=1}^{n} \delta_i \left[ f_\theta(x_i) - \log \left( \sum_{j \in R_i} \exp f_\theta(x_j) \right) \right].$$

(A5)
where \( \mathcal{R}_i = \{ j : \delta_j = 1, o_i < o_j \} \) is the at-risk set at the individual \( i \) event time \( o_i \). However, the stochastic gradient approach for negative partial log-likelihood leads to a biased estimate of the full batch gradient. Hence, instead, we take the approach that first estimates the baseline hazard function, then plugs it into the full log-likelihood, and minimizes the negative log-likelihood. Let \( \lambda_0 \) and \( \Lambda_0 \) be the estimated baseline hazard and cumulative baseline hazard functions, respectively, which are estimated solely with \( \{(o_i, \delta_i) : i = 1, \ldots, n\} \). Then, we have the following negative log-likelihood minimization problem:

\[
\min_{\theta} -\frac{1}{n} \sum_{i=1}^{n} \left[ \delta_i \left( \log \hat{\lambda}_0(o_i) + f_\theta(x_i) \right) - \hat{\Lambda}_0(o_i) \exp f_\theta(x_i) \right],
\]

(A6)

whose stochastic gradient with respect to \( \theta \) is unbiased. The quality of the full log-likelihood depends on the choice and estimation of baseline hazard functions, but we found that the results are fairly consistent with respect to the choice of baseline hazard function using neural networks with sufficiently large capacity. In this paper, we used the Weibull baseline hazard function, however, any popular baseline hazard function, including nonparametric ones such as Kaplan-Meier (Kaplan and Meier, 1958) can be used as well.

**Sampling From \( T_\theta(x) \)** In the case of the Cox PH model, the inverse CDF has a closed form if we use a parametric baseline hazard function. Thus, we use inverse transformation sampling. As CDF of \( T_\theta(x) \) is given as \( F_\theta(t|x) = P(T_\theta(x) \leq t) = 1 - \exp(-\hat{\lambda}_0(t) \exp f_\theta(x)) \), we can easily find the inverse CDF. If we use the exponential baseline hazard function \( \lambda_0(t) = \frac{1}{\rho} \), then \( T_\theta(x) \sim \text{Exponential}(\rho/\exp f_\theta(x)) \),

\[
F_\theta(F_\theta^{-1}(u|x)|x) = 1 - \exp(-\hat{\lambda}_0(t) \exp f_\theta(x)) = 1 - \exp \left( -\frac{t}{\lambda} \exp f_\theta(x) \right),
\]

and therefore,

\[
F_\theta^{-1}(u|x) = -\hat{\lambda} \log(1-u) \exp f_\theta(x) = -\lambda \log(1-u) \exp (-f_\theta(x)).
\]

To get sample \( \tau_i \) drawn from \( T_\theta(x_i) \), we first draw \( u_i \sim \text{Uniform}(0,1) \), and compute \( F_\theta^{-1}(u_i|x_i) \).

If we use the Weibull baseline hazard function \( \lambda_0(t) = \frac{\rho}{\lambda} \left( \frac{t}{\lambda} \right)^{\rho-1} \), \( T_\theta(x) \sim \text{Weibull} \left( \lambda \exp(-\frac{1}{\rho} f_\theta(x)), \rho \right) \). Even for nonparametric baseline hazard functions, for example, the Kaplan-Meier estimator, we can easily obtain the inverse function from the step function that defines the Kaplan-Meier estimator, as described in A2.

**C.3. DeepHit**

**Log-likelihood** DeepHit (Lee et al., 2018) directly models the event probability at each discretized time using a neural network \( f_\theta : x \mapsto P(T_\theta(x) = t) \). This leads to the following negative log-likelihood minimization problem:

\[
\min_{\theta} -\frac{1}{n} \sum_{i=1}^{n} \left[ \delta_i \log P(T_\theta(x_i) = o_i) - (1 - \delta_i) P(T_\theta(x_i) > o_i) \right].
\]

(A7)
Sampling From $\mathcal{T}_\theta(x)$ DeepHit defines event probability by a categorical distribution, so we can easily draw the time interval of an event from the distribution. However, sampling from a discrete probability distribution is not a differentiable operation. Thus, we used the Gumbel-Softmax trick (Huijben et al., 2022) in our implementation that allows differentiable sampling operation.

C.4. DNNSurv

Log-likelihood DNNSurv (Zhao and Feng, 2020) is a pseudo value (Andersen and Perme, 2010)-based deep survival model. It first estimates pseudo survival function for each individual using the Jackknife method with the Kaplan-Meier estimator. It is known that such pseudo value estimators are asymptotically unbiased for both censored and uncensored observations. Subsequently, they build a neural network model $f_\theta: x \mapsto P(\mathcal{T}_\theta(x) > t) \in [0,1]$ to predict the pseudo survival functions correctly. In Zhao and Feng (2020), the mean square error (MSE) is used as the loss function, but the MSE for pseudo value-based survival analysis can have convergence issues in heavily censored settings (Rahman and Purushotham, 2022). We used the modified loss function in Rahman and Purushotham (2022) and defined the problem as

$$
\min_{\theta} - \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{J} \left[ \hat{S}_i(t_j)(1 - 2P(\mathcal{T}_\theta(x_i) > t_j)) + P(\mathcal{T}_\theta(x_i) > t_j)^2 \right],
$$

where \{t_1, \ldots, t_J\} is a set of discrete time points that we define before performing Jackknife.

Sampling From $\mathcal{T}_\theta(x)$ Unlike CoxPH and DeepHit, sampling predicted survival times is not straightforward. Thus, we propose a differentiable sampling from the empirical cumulative distribution function. Given an individual, we first evaluate the empirical CDF on a grid of time points. The grid should cover the range from 0 to a sufficiently large value (e.g., the max follow-up time of the dataset). Then, we draw $u \sim \text{Uniform}(0,1)$ as in the inverse CDF sampling, find the closest empirical CDF value, then find the corresponding input value. We illustrate this procedure in Figure A2. To make the operation differentiable, we use the Gumbel-Softmin trick when we find the closest empirical CDF value. This technique not only applies to FAST-DNNSurv, but can be applied to any of the existing survival models.

C.5. Cox-Time

Log-likelihood We also consider the Cox-time model that assumes the relative risk is not only a function of covariates $x$ but also a function of $t$, that is, $\lambda_\theta(t|x) = \lambda_0(t) \exp f_\theta(t, x)$. Thus, for the Cox-time model, a neural network takes both covariate vector $x_i$ and time $t$ as inputs and outputs the log-proportional hazard, that is, $f_\theta: (x, t) \mapsto \log \lambda_\theta(t, x)$. Again, as in CoxPH model, we first estimate the baseline hazard function and plug it into the full log-likelihood (so that it is different from that of Kvamme et al. (2019)), leading to the following optimization problem:

$$
\min_{\theta} - \frac{1}{n} \sum_{i=1}^{n} \left[ \delta_i \left( \log \lambda_\theta(o_i) + f_\theta(o_i, x_i) \right) - \Lambda_0(o_i) \exp f_\theta(o_i, x_i) \right].
$$
Figure A2: Sampling From Empirical Cumulative Distribution Function. We (1) sample
$u \sim \text{Uniform}(0, 1)$; (2) find the closest empirical CDF value; (3) find the
corresponding $t$. In step (2), we substitute the Gumbel-Softmin trick for $\text{argmin}$
to make the operation differentiable.

Unlike the CoxPH model, Cox-time does not allow closed forms for relative hazard or
cumulative relative hazard functions. Thus, we use the Riemann sum to approximate the
cumulative hazard function as follows:

$$\Lambda_{\theta}(o_i|x_i) = \int_0^{o_i} \lambda_{\theta}(t|x_i)dt = \int_0^{o_i} \lambda_0(t) \exp f_\theta(t, x_i)dt \approx \sum_{j=1}^{m} \lambda_0(s_j) \exp f_\theta(s_j, x_i)(s_j - s_{j-1}),$$
(A10)

where $\{s_j : 0, \cdots, m\}$ is a properly defined grid such that $s_0 = 0$ and $s_m = o_i$.

**Sampling From $T_\theta(x)$**  No closed form CDF is available for Cox-Time. We can do this as
same as in the DNNSurv case, as described in Figure A2.

**C.6. Accelerated Failure Time Model**

**Log-likelihood**  We use the approach of Zhong et al. (2021) for the deep accelerated
failure time model, which assumes $\lambda_{\theta}(t|x) = \lambda_0(t \exp f_\theta(x)) \exp f_\theta(x)$. The minimization of
pseudo-likelihood of Zhong et al. (2021) is defined as:

$$\min_{\theta} - \frac{1}{n} \sum_{i=1}^{n} \delta_i \left[ - \log o_i + \log \left( \frac{1}{m \nu} \sum_{j=1}^{n} \delta_j \phi \left( \frac{\log o_i + f_\theta(x_i) - \log o_j - f_\theta(x_j)}{\nu} \right) \right) \right] - \log \left( \frac{1}{n} \sum_{j=1}^{n} \Phi \left( \frac{\log o_i + f_\theta(x_i) - \log o_j - f_\theta(x_j)}{\nu} \right) \right),$$
(A11)

where $\phi$ and $\Phi$ are density function and cumulative distribution function of the standard
normal distribution, and $\nu$ is the bandwidth of the Gaussian kernel (typical choice is
\( \nu = 1.3n^{-0.2} \). Moreover, its baseline hazard function is given as

\[
\hat{\lambda}_0(t) = \frac{1}{t} \left( \int_0^t \frac{1}{\nu} \phi \left( \frac{\log o_i + f_\theta(x_i) - \log t}{\nu} \right) \right)
\sum_{i=1}^n \frac{1}{\nu} \phi \left( \frac{\log o_i + f_\theta(x_i) - \log t}{\nu} \right) ds, \tag{A12}
\]

and the cumulative baseline hazard function is given as

\[
\hat{\Lambda}_0(t) = \int_0^t \hat{\lambda}_0(s) ds. \tag{A13}
\]

Finally, the survival function is given as

\[
\hat{S}(t|x) = \exp \left( - \hat{\Lambda}_0(t \exp f_\theta(x_i)) \right). \tag{A14}
\]

**Sampling From \( \mathcal{T}_\theta(x) \)**  We cannot find a closed-form CDF for the deep AFT. We can do this as same as in the DNNSurv case, as described in Figure A2.

We further note that our approach can be applied to deep extended hazard model (Zhong et al., 2021), piecewise constant hazard model (Kvamme and Borgan, 2021), and SODEN (Tang et al., 2022).

### Appendix D. Experiments (Supplement for Section 5)

#### D.1. Datasets

For all datasets, as presented in the main text, we followed the protocol of SurvSet repository (Drysdale, 2022). We imputed the missing values to the median and mode values for numerical and categorical variables, respectively. We then performed one-hot encoding for categorical variables. All covariates taking continuous values were standardized to have zero-mean and unit variance.

**FLChain**  The free light chain (FLChain) dataset resulted from a study about the relationship between serum FLC and mortality. Of the 7,874 patients, 2,169 patients (27.5\%) died, and the remaining 5,705 patients (72.5\%) were censored. It includes covariates such as age, serum creatinine, the presence of monoclonal gammopathy, etc. For this dataset, gender is the sensitive attribute.

**SUPPORT**  The Study to Understand Prognoses Preferences Outcomes and Risks of Treatment (SUPPORT) dataset was obtained from a study to understand survival over 180 days for seriously ill hospitalized patients. Of the 9,105 patients, 6,201 patients (68.1\%) died, with a median survival time of 58 days, and the other 2,904 patients (31.9\%) were censored. It contains covariates, including age, gender, education, income, physiological measurements, and co-morbidity information, etc. For this dataset, race (White, Black, Hispanic, and Asian/Other) is the sensitive attribute.

**Framingham**  The Framingham dataset collected from Framingham Heart Study (FHS) was performed to characterize major risk factors that contribute to cardiovascular disease. Of the 4,699 patients, 1,473 patients (31.3\%) died, and the remaining 3,226 patients (68.7\%) were censored. The dataset includes covariates such as systolic and diastolic blood pressure, age, body mass index (BMI), etc., where gender is the sensitive attribute.
D.2. Competitive Methods (Supplement for Section 5)

**GFDeepCox**  GFDeepCox stands for Group Fairness DeepCox, proposed by Keya et al. (2021), which aims to achieve group fairness for the CoxPH model. It encourages the average proportional hazard of each sensitive group to be similar. The problem is defined as:

$$
\min_{\theta} -\sum_{i=1}^{n} \delta_i \left[ f_{\theta}(x_i) - \log \left( \sum_{j \in R(\alpha_i)} \exp f_{\theta}(x_j) \right) \right] + \gamma \max_k \left[ \mathbb{E}[\exp f_{\theta}(X_k)] - \mathbb{E}[\exp f_{\theta}(X)] \right], \tag{A15}
$$

where $R(t)$ is the at-risk set at time $t$.

**IFDeepCox**  IFDeepCox stands for Individual Fairness DeepCox, proposed by Keya et al. (2021), which aims to achieve individual fairness for the CoxPH model. It encourages the proportional hazard to be similar for individuals who have similar inputs. The problem is defined as:

$$
\min_{\theta} -\sum_{i=1}^{n} \delta_i \left[ f_{\theta}(x_i) - \log \left( \sum_{j \in R(\alpha_i)} \exp f_{\theta}(x_j) \right) \right] + \gamma \sum_{i,j} \max\{0, |\exp f_{\theta}(x_i) - \exp f_{\theta}(x_j)| - D(x_i, x_j)\}, \tag{A16}
$$

where $D(x_i, x_j)$ is a distance function.

For GFDeepCox and IFDeepCox, we worked on top of the author’s implementation (https://github.com/kkeya1/FairSurv/).

**GFDNNSurv**  GFDNNSurv stands for Group Fairness DNNSurv, proposed by Rahman and Purushotham (2022), which aims to achieve group fairness for the DNNSurv (deep pseudo value-based survival model). In the original paper, the model is referred to as FGDP. The formulation to learn the model is:

$$
\min_{\theta} -\frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{J} \left[ \hat{S}_i(t_j)(1 - 2P(T_{\theta}(x_i) > t_j)) + P(T_{\theta}(x_i) > t_j)^2 \right] + \gamma \sum_{j=1}^{J} \left[ \mathbb{E}[P(T_{\theta}(X_k) > t_j)] - \mathbb{E}[P(T_{\theta}(X) > t_j)] \right], \tag{A17}
$$

where $\{t_1, \cdots, t_J\}$ is a set of discrete time points that we define before performing Jackknife.

**IFDNNSurv**  GFDNNSurv stands for Individual Fairness DNNSurv, proposed by Rahman and Purushotham (2022), which aims to achieve individual fairness for the DNNSurv (deep pseudo value-based survival model). In the original paper, the model is referred to as FIDP. The formulation to learn the model is:

$$
\min_{\theta} -\frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{J} \left[ \hat{S}_i(t_j)(1 - 2P(T_{\theta}(x_i) > t_j)) + P(T_{\theta}(x_i) > t_j)^2 \right] + \gamma \sum_{i,j} L \max\{0, |P(T_{\theta}(x_i) > t_l) - P(T_{\theta}(x_j) > t_l)| - \alpha D(x_i, x_j)\}, \tag{A18}
$$
where \( \{t_1, \cdots, t_L\} \) is a set of discrete time points that we define before performing Jackknife, and \( D(x_i, x_j) \) is a distance function.

Since there is no publicly available code for GFDNNSurv and IFDNNSurv, we implemented the model in our own way.

**DRODeepCox**  DRO for deep Cox proportional hazard model has been proposed by Hu and Chen (2022). We followed the authors’ implementation given in https://github.com/discovershu/DRO_COX.

### D.3. Network Structures and Optimization Details (Supplement for Section 5.1)

**Survival Model Network**  For all three datasets and for all models, we use multilayer perceptrons that have the structure: Input – Dense(64) – Dense(64) – Dense(64) – Output, where input differs by the dataset and output depends on the choice of the survival model. For instance, for the case of DeepCox Output = 1, while for DeepHit and DNNSurv, Output is the number of discretized time bins. Each dense layer is followed by batch normalization (Ioffe and Szegedy, 2015) and the ReLU activation function (Nair and Hinton, 2010), except for the output layer. For DeepHit and DNNSurv, we discretized the follow-up time into 50 bins with equal percentiles of the dataset.

**FAST Methods**  For our FAST approach, we need an additional network to estimate the mutual information of \( T_\theta \) and \( A \). For all datasets, we used multilayer perceptrons that consist of Input – Dense(32) – Dense(32) – Dense(32) – Dense(1), where the input has the shape of \( K + 1 \) because we used one-hot encoded sensitive attributes as input (where \( K \) is the number of sensitive groups). Each dense layer is followed by an ELU activation.

**Optimization**  We use Adam (Kingma and Ba, 2014) with weight decay (L2 penalty) of 0.0001 for all methods and datasets to train the networks. For all datasets, we trained the networks using batch size 256 (except for GFDeepCox, IFDeepCox, and DRODeepCox, which have to be trained with full-batch) for 200 epochs. The initial learning rate is set to 0.001, and we decreased it to 0.0001 after the first 100 epochs. Finally, we evaluated the loss function value on the held-out validation set and selected the network parameters that provided the best validation loss.
Table A1: The range of the hyperparameters of the fairness-aware models used in the experiment. Note that the grids are not wide enough to make the penalty terms dominant because the main purpose of our experiment is to encourage predictive parity as much as possible while not losing the prediction performance.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Hyperparameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>FASTDeepCox</td>
<td>$\gamma$</td>
<td>$[1, 2, 3, 5, 10, 15, 20]$</td>
</tr>
<tr>
<td>FASTDNNSurv</td>
<td>$\gamma$</td>
<td>$[0.01, 0.1, 1, 2, 3, 5, 10, 20]$</td>
</tr>
<tr>
<td>FASTDeepHit</td>
<td></td>
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</tr>
<tr>
<td>GFDNNSurv</td>
<td>$\gamma$</td>
<td>$[0.01, 0.1, 1, 2, 3, 5, 10, 20]$</td>
</tr>
<tr>
<td>IFDNNSurv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GFDeepCox</td>
<td>$\gamma$</td>
<td>$[0.01, 0.1, 1, 2, 3, 5, 10, 20]$</td>
</tr>
<tr>
<td>IFDeepCox</td>
<td>$\eta$</td>
<td>$[1.0, 0.4, 0.3, 0.2, 0.1]$</td>
</tr>
<tr>
<td>DRODeepCox</td>
<td>$\eta$</td>
<td>$[1.0, 0.4, 0.3, 0.2, 0.1]$</td>
</tr>
</tbody>
</table>
Appendix E. Additional Experimental Results

Figure A3: The predicted survival curves generated by DeepCox-based and DNNSurv-based fairness encouraging methods for the FLChain dataset.
Figure A4: The predicted survival curves generated by DeepCox-based and DNNSurv-based fairness encouraging methods for the Framingham dataset.
Figure A5: Three performance metrics (CI, Brier, and ECE) vs. GF trade-off curves from FLChain dataset.

Figure A6: Three performance metrics (CI, Brier, and ECE) vs. GF trade-off curves from Framingham dataset.
Figure A7: Three performance metrics (Ci, Brier, and ECE) vs. GF trade-off curves from SUPPORT dataset.
Figure A8: Results of FLChain dataset where the sensitive attribute is gender.
Figure A9: Results of Framingham dataset where the sensitive attribute is gender.
Figure A10: Results of SUPPORT dataset where the sensitive attribute is race/ethnicity.
Figure A11: Follow-up time distributions for different max follow-up times for male participants (FLChain).
Figure A12: Follow-up time distributions for different max follow-up times for male participants (Framingham).
Figure A13: Model performances and disparities versus max follow-up years for males on the Framingham dataset averaged over 5 test sets. The green shades represent the 5% margin of the C-index.

Figure A14: Model performances and disparities versus max follow-up years for males on the Framingham dataset averaged over 5 test sets. The green shades represent the 5% margin of the ECE.
Figure A15: Model performances and disparities versus max follow-up years for males on the FLChain dataset averaged over 5 test sets. The green shades represent the 5% margin of the C-index.

Figure A16: Model performances and disparities versus max follow-up years for males on the FLChain dataset averaged over 5 test sets. The green shades represent the 5% margin of the ECE.
References – Supplementary


